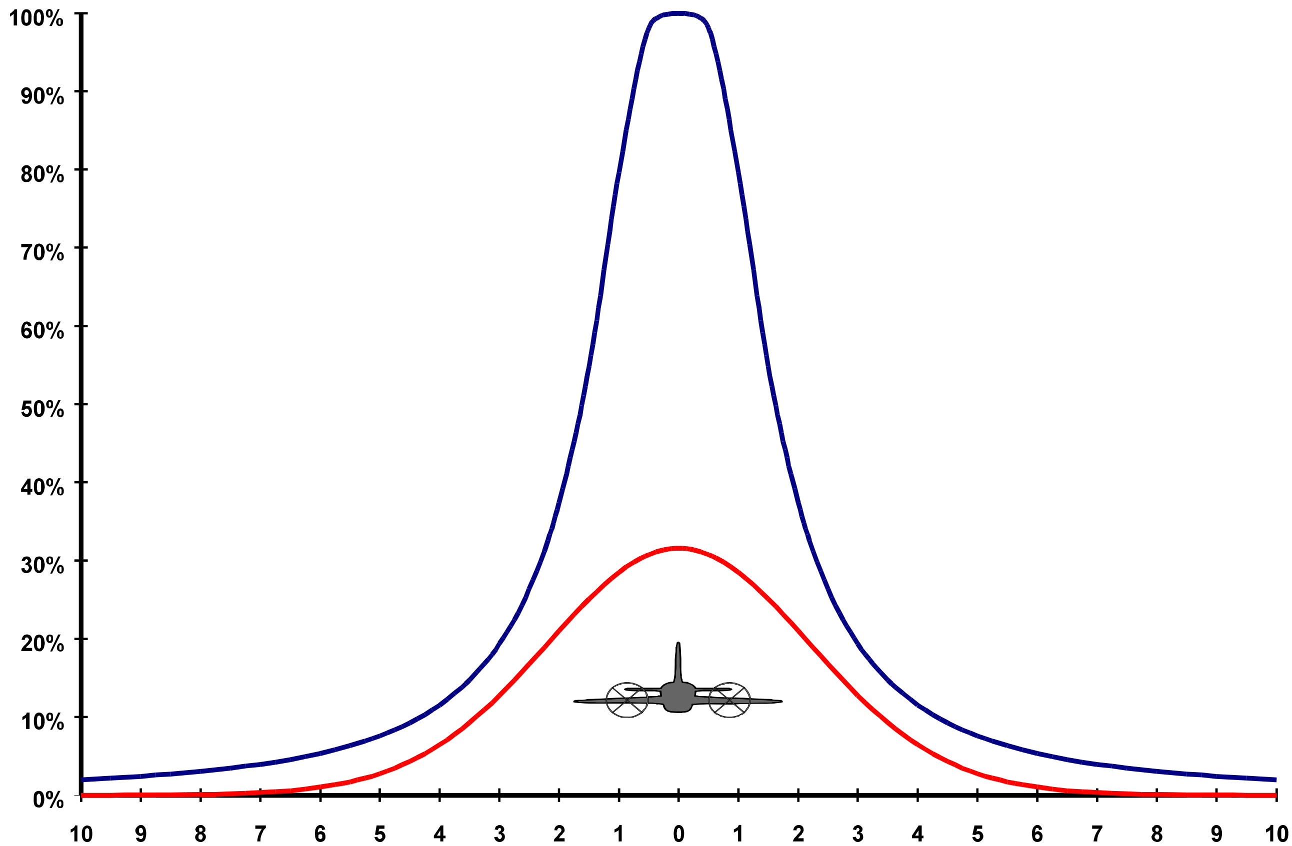
The Theory of Search

A Simplified Explanation



Soza & Company, Ltd.

and

Office of Search and Rescue
U.S. Coast Guard

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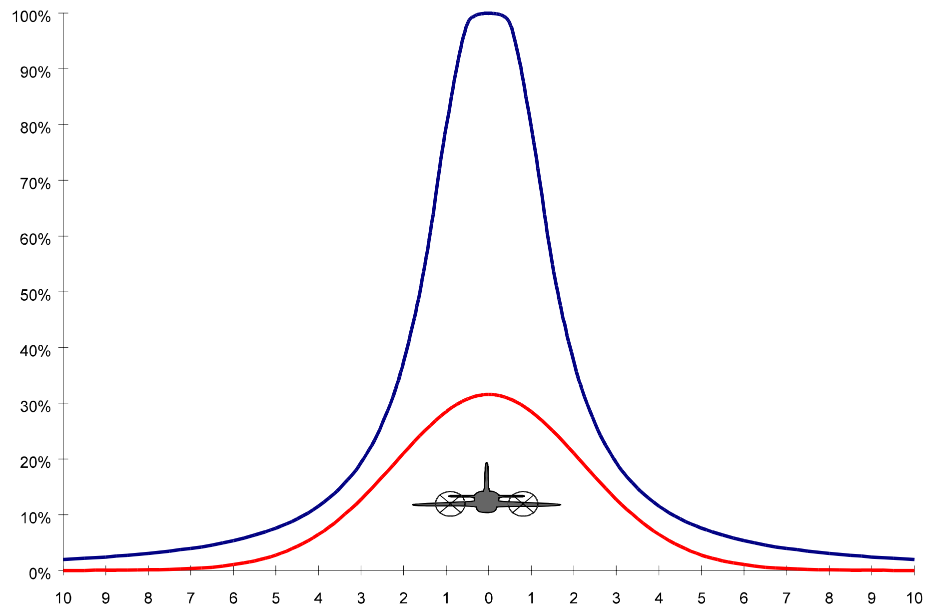

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The Theory of Search: A Simplified Explanation
Table of Contents

Chapter 1
Introduction

1.1 Introduction.The purpose of this paper is to provide the search and rescue (SAR) community with a single document explaining the scientific basis for search planning. Both the *Simplified Search Planning Method* (SSPM), in place for the last 50 years or so, and recently proposed refinements collectively called the *Improved Search Planning Method* (ISPM), share this basis. The only difference between the two methods is that the ISPM is a more complete and flexible implementation of basic search theory than the SSPM. The main body of this paper explains the basic theory of search. In the last chapter, a comparison is made between the SSPM and ISPM to show which method produces search plans having the best chance of succeeding.

1.1.1 This document was prepared by Mr. J. R. Frost of Soza & Company, Ltd., of Fairfax, Virginia, U.S.A., in cooperation with the U.S. Coast Guard.

1.1.2 The SSPM is based upon a number of specific assumptions about the search object's probable location, the nature of visual detection and the way in which searches are conducted. These assumptions, which are explained later in this paper, are:

- The possible search object locations are distributed around a datum position in a circular normal probability distribution;
- The means of detection is visual;
- The inverse cube model of visual detection (which is based on its own set of assumptions and the geometry of instantaneous detection opportunities from aircraft) is sufficiently accurate under all search conditions;
- Searches are performed as series of equally spaced parallel sweeps relative to the search object; and
- Specific levels of *coverage* or *search effort* are used for each search in a series of searches for a search object.

1.1.3 Some refinements have recently been recommended as a more complete implementation of the basic theory to produce the ISPM. These refinements include

- Increased flexibility allowing optimal search plans to be developed for any level of available search effort;
- Extensions (discussed only briefly in this paper) for use with probability distributions other than the circular normal type; and
- Extensions (also discussed only briefly) to accomodate varying search conditions.

1.1.4 Most of the material in this paper is based on the work of B. O. Koopman as documented in his book Search and Screening [1]. However, while Koopman developed the general theory of search, a number of specific assumptions, some listed above, were apparently made in the application of this theory to the development of the SSPM. Documentation of the rationale for these assumptions and the identity of the original developers of the SSPM seem to have been lost. This paper endeavors to provide a simplified explanation of Koopman's theoretical work and show how it has been applied to SAR, including speculations about the rationale behind some of the apparent assumptions made when the theory was translated into practice. This paper will attempt to explain the theory of search in sufficient detail to provide the general reader with a practical understanding of the subject, but the level of the mathematics used will be kept to the minimum required to achieve a pragmatic appreciation of the necessary concepts. Mathematical rigor may be found in the references provided in the bibliography for readers who require it.

1.1.5 The discussion of search theory will proceed as follows. The remainder of this chapter will be devoted to describing the major characteristics of searching, giving some non-SAR examples of common search situations, and placing search operations in their proper context with respect to the total operation aimed at achieving a particular goal. Chapter 2 develops the three the basic probability concepts that are central to search theory. Chapters 3, 4, and 5 then develop each of these concepts in some detail. Chapter 6 brings the three central probability concepts back together to explain the theory of optimal search. Finally, although the mechanics of the SSPM and ISPM are not discussed in detail, a comparison of the results obtained with each is presented in Chapter 7.

1.2 Practical Aspects of SAR and Searching. Before any rescue operation can take place, the survivors must first be found. In many cases, searching is limited to locating the survivors in a very small area based on a position provided by the survivors or signals from the survivors' location (such as those from an emergency position indicating radio beacon or EPIRB) from which a position can be deduced. Other efforts are intended to "take the *search* out of search and rescue," e.g., by maintaining and improving the Cospas-Sarsat system and increasing the availability of devices such as EPIRBs through cost reduction and, in some cases, carriage regulations. However, electronic emergency beacons will not always be where they are needed, may not always survive the SAR incident in good working order, may become separated from the survivors, and will always have a limited active life due to the necessity of being battery powered. Although the relative number of SAR incidents where significant searching is required may go down, it is highly unlikely that the need for searching can be completely eliminated in the foreseeable future. Searching will remain an essential element of SAR for some time to come.

1.2.1 Searching becomes necessary when it is known, or there is sufficient concern, that a SAR incident has occurred but there is a large uncertainty as to the location(s) of the survivors. Sometimes investigative efforts can uncover clues which substantially reduce the uncertainty about where the survivors are. For this reason, SAR Mission Coordinators (SMCs) should pursue investigative efforts vigorously from the moment of the first alert until the survivors are actually located and rescued. However, in many situations, especially those involving survivors adrift on the ocean, the uncertainty in the survivors' location can increase rapidly with time, making the search problem increasingly difficult. In addition, the chances of the distressed persons' continued survival often decrease rapidly with time. For these reasons, searching must often begin early in the case based on the available clues, however large their uncertainties may be, if a rescue is to be effected.

1.2.2 Searching is a complex, arduous, time consuming, and expensive task. These characteristics provide ample justification for trying to "take the *search* out of search and rescue." However, these same characteristics also justify expending a good deal of effort in the planning stages to make those search efforts which are necessary as efficient as possible. In the long run, greater efficiency will reduce the amount of search effort required to obtain successful results and will often reduce the time required to locate the survivors. That is, better search planning methods can take some of the *search effort* out of SAR and increase the number of lives saved at the same time.

1.2.3 Everything in the preceding paragraphs has been self-evident to SMCs and search planners for many years. What has been much less evident is that the *operation of search* actually has a considerable scientific foundation. The SSPM, as published in various national and international SAR manuals, contains almost no hint of its scientific background. On the other hand, the scientific work that has gone into the development of devices (and the platforms that carry them) to detect and identify objects from increasingly large distances has been widely publicized. While the unaided human senses, search facilities such as aircraft and vessels, and various detection and navigation devices and aids are *necessary* for a successful search, they are not *sufficient*. The manner in which these components are employed in a search effort is just as important as the individual capabilities of the components themselves. It was this realization which started the development of search theory as a branch of applied science.

1.3 <u>Practical Examples of Search Activity</u>. Searching as an activity is not something that is unique to SAR. It is actually a common activity undertaken for a wide variety of purposes in an equally wide variety of settings. The non-SAR examples in the subparagraphs below may help to put the general problem of search in perspective.

1.3.1 We all search for something almost every day. Such searches may be for misplaced objects such as keys, papers, jewelry, etc., or the lavatories in a strange building, or any of hundreds of other things whose exact location is not immediately known. Usually we try to think of the most likely location and concentrate our search effort there, until either the object is found or continued lack of success makes us wonder if we are looking in the right place. As we will see later in this paper, this intuitive approach to search planning is supported by the mathematics of search theory. Another thing daily experience teaches is that an organized pattern of search is more likely to succeed than casting about randomly, especially if the object is small or blends with its background.

1.3.2 Archeologists search for lost cities by looking for clues in ancient writings and records, examining the geology of the region, interviewing indigenous peoples, and then physically searching for evidence at the site where the ancient city is thought to have stood. In other words, a substantial initial investigative effort is made to reduce the uncertainty about the ancient city's probable location. Once the search area is reduced to a manageable size, a carefully planned and organized physical search is undertaken for further evidence. This is basically how the ancient city of Troy, once thought to be purely fictional, was found in the last century, over 3,000 years after the Trojan War described in Homer's Iliad. Investigation is an important activity in the search planning process for the same reason — to reduce uncertainty about the survivors' location.

1.3.3 Mining and oil companies search for mineral and petroleum deposits which may be profitably exploited. Prospectors for such deposits look first for large-scale geologic features which indicate the minerals they are seeking are likely to be nearby. When one is found, they often detonate small amounts of carefully placed explosives in the vicinity, then record and analyze the seismic consequences. This allows them to reduce the uncertainty about whether deposits are present. If there are indications of minerals or oil, the seismic data may also provide indicators about the size of the deposit as well as a better (less uncertain) estimate of its exact location. If the results of the seismic "search" are encouraging, then a more detailed, and expensive, search may be undertaken in the form of drilling, tunneling, or limited excavation. While each of these steps tends to reduce the uncertainty about whether a deposit exists and can be mined, uncertainty about whether a profit will be made often remains significantly high for quite some time after mining operations have begun.

1.4 Search Object Detection and Recognition. To succeed in finding an object, it is necessary to both detect the object and recognize it as the object being sought. When the object is in plain view and sufficiently close to the observer, its recognition is so immediate that detection and recognition seem to happen as a single act. In most search problems, however, recognition can easily be a matter of real difficulty apart from detection. Detection occurs when the searcher *perceives* a set of sensory impressions indicating the presence of an object. Recognition requires *interpretation* of these impressions to determine their source or cause. With detection equipment, what is perceived may be "blips" on a radar screen or sounds in an ear-phone and it is necessary to interpret these clues, in light of all other available information, to decide whether they are caused by the object of the search.

1.4.1 Three factors are involved in the act of recognition:

1. Factors of a physical order, such as the nature and properties of the search object, those of the detecting equipment, and those of the environment.
2. The time and place of the detection, capabilities and likely behavior of the survivors, and any correlation with recent reports or events.
3. The psychological capability of humans to "recognize" — as in the recognition of an individual face.

1.4.2 Electronic sensors typically perform only the detection function although in some cases, primarily of a military nature, electronic or sonic "signatures" can be used for identification purposes. For all practical purposes, electronic sensors used in SAR searches do not perform the recognition function. One exception is the 406 MHz EPIRB. Beacons of this type emit a data stream to orbiting satellites which includes a unique serial number and possibly a code indicating the nature of the incident. If the beacon has been properly registered, information about the owner, craft and lifesaving equipment can be retrieved from a database. However, search facilities on scene only receive a 121.5 MHz homing signal that contains no recognition data.

1.4.3 The detection capabilities of electronic sensors are constantly being improved so that smaller and smaller objects may be detected at greater and greater distances. However, we must not lose sight of the fact that the act of *recognition* is essential. Sometimes electronic information alone can be used to determine that an object is *not* the search object. The International Ice Patrol uses a Side-Looking Airborne Radar (SLAR) to detect, record on film, and plot the locations of icebergs. However, SLAR detects ships as well as icebergs. Detections of ships are eliminated by computing the course and speed of each detected object from the SLAR data. (Overlapping sweeps are used so that each object is normally detected at least twice during a patrol.) Course and speed information allows the SLAR operators to separate vessels from icebergs without actually sighting either. Similar techniques can sometimes be used in SAR searches. However, in SAR, recognition often requires visual inspection of each detected object to determine whether it is the search object. This fact, and the fact that most SAR search objects are difficult to detect electronically, are the primary reasons the human eye remains the sensor of choice in SAR searches.

1.5 <u>Search Object Motion</u>. In subparagraph 1.2.1 above, it was observed that common experience teaches us that organized searching usually produces better results than random, disorganized efforts. However, the objects of many common, everyday searches are stationary and do not move as the search progresses whereas the objects of SAR searches are often in motion. Survivors adrift on the ocean move with the winds and currents. Survivors on land often move to find shelter or improve their circumstances in some other way. When organizing a search, it is important to estimate the motion of the survivors and account for it in determining both *where* to search and *how* to search. The frame of reference for any regular pattern of search must be fixed on the search object or at least the best estimate of its location and movement during the search effort. Otherwise, the results can easily be even worse than those of random search.

1.6 Searching as Part of a SAR Response. When the SAR system is alerted that a SAR incident has, or may have, occurred, the information contained in the initial alert is usually incomplete. The alert may originate from a concern that a SAR incident *may* have occurred, making the very existence of the incident uncertain. Even when it is known that a SAR incident has occurred, the time, place, and nature of the incident may not be precisely known. Many other important pieces of information may be either missing altogether or known only within very broad limits. All this means the SMC is often initially faced with a mystery having few clues and some of those may be irrelevant, unreliable, or both.

1.6.1 The way to solve a mystery, of course, is through investigative efforts; and investigative efforts generally proceed via a process of elimination. Most investigations follow a pattern like that shown below.

1. Evaluate the initial clues.
2. Develop initial theories, or scenarios, consistent with the clues, about what may have happened or what may be true.
3. Actively seek more clues.
4. Evaluate and rank all clues found to date according to relevance and reliability.
5. Eliminate previous scenarios when they are no longer consistent with the accumulated body of evidence. Develop new scenarios, if needed, which are consistent with the accumulated body of evidence.
6. Return to step (3) until the mystery is solved.

1.6.2 One way to actively seek more clues is to search those areas where the search object may be located. Searches can provide either "positive" or "negative" clues.

1.6.2.1 When actual evidence of the SAR incident or subsequent survivor movement is found, then the clues are "positive." For example, finding debris that can be identified as belonging to a missing vessel is a positive clue that a SAR incident has occurred. Depending on the discovery, such evidence may also provide positive clues about the time, place and nature of the incident.

1.6.2.2 Searching an area without finding anything is a "negative" clue. If the search was 100% effective, then it is clear that the search object was not in the area at the time of the search. This information may allow some previous scenarios to be eliminated. If the search object is known to be stationary, such results also eliminate the need to search the area again at a future time. Usually, searches are not 100% effective and so they only reduce, rather than eliminate, the likelihood of the search object being in the area when it was searched. Even so, such a reduction is an indicator that perhaps future search efforts would be better expended elsewhere. Exactly how search effort should be allocated will be discussed in Chapter 6.

1.6.3 Although searching can be the most expensive part of a SAR response, involving the most people and equipment, it must be viewed as only a portion of a larger operation. Other investigative efforts besides active searching should continue unabated. All of the available clues and information should be reviewed and re-evaluated at least daily. Plans need to be made for the next search effort in case the current effort fails. Other activities, as well as the search itself, need to be coordinated. Searching is really just one more investigative tool used to locate survivors. It is needed when other techniques have either failed or are not expected to produce results as early as may be necessary to save lives.

Chapter 2
Basic Probability Concepts

2.1 The Role of Probability in Search. Every SAR case involving a search is beset with uncertainties. At a minimum, the survivors' location is uncertain; otherwise, there would be no need to search. Usually there are uncertainties in other important quantities, including such things as:

- when, where and possibly whether a SAR incident actually occurred;
- the direction and speed of survivors' movements since their locations were last known;
- the size and other characteristics of the search object affecting the ability of search facilities to detect it; and
- environmental factors affecting search object motion, search facility sensor performance, and survivor behavior.

These are just a few of the factors the search planner must take into account. The nature and impact of these uncertainties on search planning and operations can be understood quantitatively only in terms of the mathematical discipline known as probability theory.

2.2 Elements of Successful Searches. For any search to be successful, two things must be true. First, searchers must be looking in the right place. Second, searchers must be capable of detecting the search object. Since the exact location of the search object is never known in advance, "looking in the right place" means searching areas that have at least some *probability of containing* the search object. Similarly, the searchers must be using sensors that have at least some *probability of detecting* the search object. If either of these probabilities is zero, the search is doomed to fail. Only if both probabilities are 100% is the search guaranteed to succeed. In actual operations, the *probability of success* lies somewhere between these extremes. In other words, the probability, or likelihood, that a search will succeed in locating the search object depends on two other probabilities:

(1) the probability that the search object is actually in the area searched; and
(2) the probability of detecting the search object if it is there.

Each of these in turn depends on a number of other factors. For example, the ability to detect the search object depends on such things as the type and characteristics of the sensors used, the environmental conditions in the search area, the way the sensors are employed, etc. The likelihood of the search object being in the search area depends on the amount and accuracy of the information available about the circumstances of the case, the size and location of the search area, etc. These details will be discussed further below. At this point in the discussion, it is sufficient to realize that success depends on both sensor performance and sensor placement. In the most general terms, the objective of search planning is to place the available sensors in the areas where the search object is likely to be.

2.3 Probability of Containment. *Probability of containment* (POC) measures the likelihood of the search object being contained within the boundaries of some area. It is always possible to achieve a 100% probability of containment by just making the area larger and larger until all possible locations are covered. However, in practice, search facilities are usually limited and so the amount of area which can be effectively searched is also limited. For this reason, it is important for search planners to use all the available evidence about the SAR incident to do the following:

- Eliminate as much of the earth's surface as possible from the *possibility area*. That is, determine the smallest area, consistent with all the available facts, which contains all possible survivor locations. By this definition, the initial POC for the possibility area is 100%;
- Within the possibility area, estimate which sub-areas are more likely to contain the survivors and which are less likely based on the available information; and
- Quantify these estimates by assigning numeric POCs to the sub-areas. Initially, the sum total of all sub-area POC values should be 100% for the entire possibility area.

Methods for estimating POCs and developing *probability maps* are discussed in Chapter 3.

2.4 Probability of Detection. When performing a search under actual operational conditions, detecting and recognizing the search object is by no means a certain outcome. (From this point forward, the word "detection" when used without qualification will be taken to mean true detection with identification.) This is particularly true of SAR search objects which tend to be small and often do not contrast well with their surroundings. Such objects may be passed quite closely without being detected, even by vigilant searchers.

2.4.1 Typically, the likelihood (probability) of detecting an object decreases with its distance from the searcher. Three specific detection models will be explored in some detail in Chapter 4. For now, we will discuss how probability of detection values should be interpreted for purposes of search planning and evaluation.

2.4.2 *Probability of detection* (POD) is a measure of sensor performance in a particular search or, alternatively, a measure of how well an area has been searched.

2.4.2.1 Under the first interpretation, POD describes the ability of a particular sensor to detect a particular type of search object under a given set of operational and environmental conditions. For example, the POD for visual search from an aircraft flying a parallel track search pattern at 120 knots with a 3 nautical mile spacing between adjacent tracks at an altitude of 500 feet over the ocean in daylight in clear weather searching for an eight-person liferaft might be 75%. (Detection models are discussed in Chapter 4 and exactly how specific values for POD are estimated is discussed in Chapter 5.) As a measure of sensor performance, POD is independent of whether the object of a particular search is in an area covered by the sensor. In other words, POD only describes the ability to detect the search object ***if it happens to be in the area searched***. This is called a *conditional probability* in the terminology of probability theory. A conditional probability has little value except when used in combination with the probability that the condition on which it is predicated is true. In the above example, the chances of the condition (liferaft in the area searched) being true are not known, so the chances of finding the liferaft during that particular search are also unknown.

2.4.2.2 From the above example, it is easy to understand the second interpretation of POD as a measure of how well an area has been searched. If an area was searched under the circumstances described in the example, then there was a 75% probability (three chances in four) that an 8-person liferaft would be detected if one was in the area at the time the area was searched.

2.4.3 It is important to realize that POD alone is **not** a valid measure of the search's chances for success. It only measures how well a sensor performs or how well an area was searched.

2.5 Cumulative POD. If the same area is searched multiple times, the chances of detecting any search object in the area are increased. The probability of detecting an object during at least one of the two searches is one minus the probability of failing to detect it on both searches. Probability theory tells us that the chances of two independent events both happening is the product of their probabilities. For example, the probability of flipping a coin and having it land with a particular side facing up is one chance in two or 0.5. The chances of flipping it twice and having the same side facing up both times is 0.5 x 0.5, or 0.25. If the probability of detection for a search is 0.6, then the probability of failing to detect (*PFail*) is 1 - 0.6 or 0.4. If the POD on a second search of the same area is 0.7, then the *PFail* is 1 - 0.7 or 0.3. The probability of twice failing to detect an object that is there to be found on these two successive searches is the product of their two *PFail* values or 0.12. Therefore the cumulative POD for the two searches is 100% minus the probability of failing to detect on either of the two searches. That is, 1 - 0.12 = 0.88 (88%). The following formula states the general principle in compact form,

[2-1]
$$POD_c = 1 - (1 - POD_1)(1 - POD_2)$$

where POD_c is the cumulative POD after two searches of the same area, POD_1 is the POD of the first search and POD_2 is the POD of the second search. This formula may be further generalized to accommodate any number of successive searches.

2.6 Probability of Success. In paragraph 2.2 it was stated that the *probability of success* (POS) for a particular search depends on both the probability of the search object being in the area searched and the probability of detecting the search object if it is there. In other words, POS depends on both POC and POD.

2.6.1 Probability theory tells us that the probability of two events in the same “sample space” both happening is the product of the two event probabilities. In searches, the “sample space” is the search area during the time when sensors are present and the two events are

A = the search object being in the area searched; and
B = the sensor detecting the search object.

In the notation of probability theory

[2-2] $$P(A \cap B) = P(A) x P(B|A)$$

where

$P(A)$ represents the probability that event *A* (the search object being in the search area) is or will be true (POC);

$P(B|A)$ (read probability of event *B* happening assuming event *A* is true) represents the conditional probability that event *B* (finding the search object) is or will be true provided the object is in the search area (POD); and

$P(A \cap B)$ (read probability of both *A* and *B* being true) represents the probability of finding the search object during the particular search being evaluated (POS).

2.6.2 Using more familiar notation,

[2-3] $$POS = POC x POD$$

That is, to find the probability of success for any search, simply multiply the probability of the search object being in the area searched by the probability of detecting the search object if it is there.

2.6.3 Predicted POC and POD values may be used to predict the POS for a planned search. Estimates of actual POC and POD values, based on actual conditions in the search area, the latest analysis of all available information, the amount of the intended search area actually covered, etc., may be used to estimate the actual POS of a search anytime after it has been completed. It is important to realize that the actual POS can never be known — only estimates of its value are possible and these may change with the passage of time as more information comes to light. For example, if a new clue is found following a search which indicates the original estimate of a search area's POC was too high (or too low), then a new estimate of that area's POC should be made and the associated POS value, and all subsequent POS values, should be recomputed, even if several days have passed between completing the search and finding the new clue.

2.7 Importance of POS. The importance of POS is that unlike either POD or POC standing alone, POS ***is*** a valid measure of a search's chances for success. That is, it is a valid measure of *search effectiveness*.

2.7.1 Earlier, it was stated that the objective of search planning is to place the available sensors in areas where the search object is likely to be. Now this goal may be stated in more precise terms.

The goal of search planning is to maximize the probability of success.

Knowing this goal provides an opportunity for comparing alternative search plans and an explicit, computable criteria for deciding which of them is best. For example, if a search planner develops three different but attainable plans for a search and predicts their POS values to be 40%, 55%, and 50% respectively (based on the predicted POC and POD values for each plan), then the second plan (POS = 55%) is the one which should be used.

2.7.2 While the second search plan may be the best of the three alternatives listed above, it still may not be the best possible search plan. In other words, it may be possible to develop another search plan, using the same search facilities looking for the same search object under the same environmental conditions, which has a predicted POS greater than 55%. A search plan which produces the maximum possible POS value using the available search facilities is said to be *optimal*. Using the available search facilities in a way that produces the maximum possible POS is called *optimal effort allocation*. We will return to these concepts and explain them in more detail in Chapter 6.

2.8 Adjusting POC. There is another important relationship between POC and POD besides POS. When an area is searched but the search object is not found, the probability that the search object was in the area when it was searched is reduced by an amount proportional to the POD. For example, if the POC of an area was 60% before searching and it was then searched with a POD of 75%, the POC after searching would be only 15%. Stated as a formula,

[2-4] $$POC_{after} = POC_{before}\, x\left(1POD\right)$$

It is important to adjust POC values after each search. Determining where to place the effort for the next search, predicting the POS for the next search, and computing the cumulative POS for all searching done to date all depend on updating the POC values to reflect the results of previous searching. More details on how to update estimated POC, POS and cumulative POS values are presented in Chapter 5.

2.9 <u>Cumulative POS</u>. Cumulative probability of success (POS_C) measures the likelihood of having found the search object based on the results of all searching done to date. Consider the following situation. The search object is known to be stationary. An area with an initial POC of 0.75 (75%) is searched with a POD of 0.6 (60%). A second search of the same area is done with a POD of 0.7 (70%). In this situation, there are three ways to compute cumulative POS.

2.9.1 When the same area is searched twice and there are no influences on the search object location probability distribution other than the searches themselves, the cumulative POD may be used to compute the cumulative POS. Using equation [2-1], the value of POD_C for this example is computed as,

$$POD_c = 1\left(10.6\right)\left(10.7\right) = 0.88$$

or 88%, and using a slight modification of [2-3] to compute POS_C,

$$POS_c = POC x\, POD_c = 0.75x0.88 = 0.66$$

or 66%. **This method of computing POS_C is NOT recommended because it is valid only under a very restrictive set of conditions which do not normally occur in actual operations.**

2.9.2 The second, and recommended, method for computing POS_C, and one which works in all situations, is to compute the POS for each search, being sure to adjust the POC value for the search area to account for previous searching. The cumulative POS will just be the sum of the individual POS values. That is,

$$POS_c = \sum_{\substack{All \\ Searches}} POS$$

[2-5]

For example, using the same POC and POD values given above, the POS the first search is

$$POS_1 = POC_0 \, x \, POD_1 = 0.75x0.6 = 0.45 \; ,$$

or 45%. The POC for the area following this search is

$$POC_1 = POC_0 (1 - POD_1) = 0.75x0.4 = 0.3 \; ,$$

or 30%. Computing the POS for the second search

$$POS_2 = POC_1 \, x \, POD_2 = 0.3x0.7 = 0.21 \; ,$$

or 21%. Adding the two POS values to get the cumulative POS,

$$POS_c = POS_1 + POS_2 = 0.45 + 0.21 = 0.66 \; ,$$

or 66% as before. If searching is thought of as a way to remove, or subtract, probability from a distribution, then after the second search it could be said that of the original 100% probability in the possibility area before searching began, 66% has been removed, leaving 34% behind. Conversely, if it was known that the amount of probability remaining was 34%, it could be concluded that the value of POS_C up to that point was 66%. This leads to the third method for computing POS_C.

2.9.3 The third method for computing POS_C, which may also be used under any set of circumstances, is to adjust the POC of each area searched immediately following the search and use the updated POC as the entering argument for the next update. After all the POC values have been adjusted for all searches, the total probability remaining in the possibility area can be subtracted from 1.0 to get the cumulative POS. For example, using the same POC and POD values given above, the POC for the area following the first search is

$$POC_1 = POC_0 (1 - POD_1) = 0.75x0.4 = 0.3 \; .$$

That is, the POC of the area after the first search is 30%. For the second search,

$$POC_2 = POC_1 (1 - POD_2) = 0.3x0.3 = 0.09 \; .$$

Now the POC for the area that has been searched twice is down to 9%. By definition, before any searching was done, the *possibility area* had a POC of 100%. The area that was searched contained only three-fourths of the total amount of probability or 75%. This means that 25% of the possible search object locations were never searched at all. So, the total POC for all areas outside the search area is still 25%. Adding this value to the 9% remaining in the search area produces a total POC for the original possibility area of 34%. Subtracting this from 1.0 gives 0.66 or 66% just as in the other two examples. Stated mathematically,

$$POS_c = 1 - \sum_{\substack{All \\ Subareas}} POC$$

[2-6]

In other words, if the all the current POC values, properly adjusted to reflect the results of all previous searching, are added together and subtracted from 1.0 (100%), the result will be the current value of POS_C. In computing the sum of all POC values, all subareas must be accounted for, regardless of whether or not they have been searched. This concept will be revisited in Chapter 6.

2.10 Importance of POS_C. Consider a situation where searchers have been looking for a search object for several days. At the end of the first day, the cumulative POS might be 60%, at the end of the second day it might be 80%, and at the end of the third day it might be 90%. Cumulative POS is important for the following reasons.

2.10.1 First, POS_C provides an indication of whether the search effort is being used in the right place. Achieving a high estimated cumulative POS value without finding the search object may mean the searchers are looking in the wrong place. All of the available information and clues should be carefully reviewed and analyzed in light of the continued lack of success to determine whether the search effort should be relocated.

2.10.2 Second, cumulative POS provides an indicator of whether searching should continue. If the search planner is certain that the searchers are looking in the right place and the POS value is very high, then the chances of finding the survivors with further searching is correspondingly small. If the POS_C value is estimated to be 99%, then no matter how much more search effort is expended, the chances of finding the search object on a subsequent search must be estimated as no greater than 1%. Suspending active search operations is always a difficult decision which must be based on many factors. POS_C should be one of the factors considered.

2.11 Historical Perspective. For many years, only one of the three probabilities (POC, POD, and POS) discussed above appeared in search and rescue manuals. POD and methods for estimating it for visual search based on the size and type of the search object, the height of the observer, and on scene environmental conditions appeared in search and rescue manuals, but neither POC nor POS were mentioned.

2.11.1 The absence of POC and POS from the published search planning techniques does not mean these two probabilities were either unknown or unimportant to the developers of those techniques. In fact, POC and POS actually played a large role in the development of the simplified search planning method (SSPM), which has been used for many years. Certain assumptions were made about the nature of the distribution of search object location probabilities (from which the assumed POC values of search areas could be deduced), how searches would be conducted, and the PODs of the successive search efforts. An SSPM which produced optimal search plans (with maximum POS values) when all the assumptions were true was then developed and published for the SAR community's use.

2.11.2 Unfortunately, the concepts of POC, POS and the goal of maximizing POS were not presented to the SAR community in general. Even worse, the assumptions about POC, POD, and the conduct of searches on which the published SSPM was based often did not match the practical realities faced by search planners. Finally, the SSPM provided no flexibility or guidance for dealing with situations where its underlying assumptions were not true. As this paper progresses, the assumptions on which the SSPM is based are pointed out and the method's limitations discussed.

Chapter 3
Probability Density Distributions and Probability Maps

3.1 Introduction. One of the first things to be determined when planning any search is *where to search*. A first approximation of where to search is the *possibility area*. Further refinements of the best area to search depend on how the search object location probability density is distributed within the possibility area, how much search effort is available, and how that effort is applied. This chapter concentrates on search object location *probability distributions*. Chapter 5 discusses the concept of *search effort* and Chapter 6 discusses the planning of *optimal searches*.

3.1.1 The possibility area is found by eliminating as much of the world as possible from consideration so that only those locations where the survivors could be, based on the available information, are included. Searching outside the possibility area clearly makes no sense since the POC is zero and hence the POS would also be zero. However, searching the entire possibility area also may not make sense if there are insufficient numbers of search facilities to cover it effectively or if some parts of the possibility area are significantly more likely to contain the survivors than other parts. In the latter case, the *probability* is not uniformly *distributed* over the possibility area. Intuitively, it would seem wise to concentrate the search effort in those areas where the *probability density* (amount of probability per unit area, or POC divided by the area) is highest. This is often, but by no means always, the case. Chapter 6 discusses how to determine the optimal area to search.

3.1.2 Dividing the possibility area into subareas and assigning POC values to each is often difficult. Attempts to complete such a task with no guidance could easily degenerate into pure guesswork. Fortunately, there are some standard probability density distributions which can be used as guideposts. In the paragraphs that follow, we will examine some probability density distributions often used to describe a search object's probable locations. The purpose is to become familiar with the major characteristics of these distributions. Such familiarity is necessary for understanding the theoretical basis for the search planning techniques recommended in practice.

3.2 Datums. In surveying, map making and other endeavors involving measurements, it is often necessary to establish a *datum* or reference to be used as a basis for making the measurements. In land surveying, a precisely known location, or datum, is used as a starting point for the survey and all distances in the vicinity are referenced to that datum. Elevations and depths on most charts and maps use mean sea level as a datum. In search planning, datums are used as references for determining where to search. Such datums may be represented by points, lines, or areas, depending on whether the SAR incident is known or believed to have occurred in the vicinity of a specific location, in the vicinity of a line, or somewhere in an area. Since there is always at least some uncertainty about the search object's actual location, these datums are actually references for probability density distributions.

3.3 Point Datums. When the SAR incident is known or believed to have occurred in the vicinity of a specific location or position, a point datum is used. The assumed probability density distribution for a point datum is the *circular normal distribution*. This a three-dimensional version of the standard normal distribution (or "bell curve") found in any elementary probability or statistics text. When a group of points (x_1,y_1), (x_2,y_2),...,(x_n,y_n) is distributed in such a way that the x_i have a normal distribution, the y_i also have a normal distribution and the standard deviations of the two distributions are the same, then a circular normal distribution results, and vice versa.

The Simplified Search Planning Method (SSPM) assumes a circular normal distribution of possible search object locations.

For this reason, the circular normal distribution will be examined in detail below.

3.3.1 Figure 3-1 illustrates the circular normal distribution. The grid represents *x* and *y* co-ordinates in the "plain" while the height of the "mountain" at any point represents the probability density at that point. The POC value for any circle centered on the datum point (or peak of the mountain) is found by

[3-1] $$POC = 1 e^{\frac{R^2}{2}}$$

where *R* is the radius of the circle in standard deviations and *e* is the base of the natural logarithms. If *R* is one standard deviation, then the POC is

$$POC = 1 e^{\frac{1}{2}} = 10.61 = 0.39$$

or 39%.

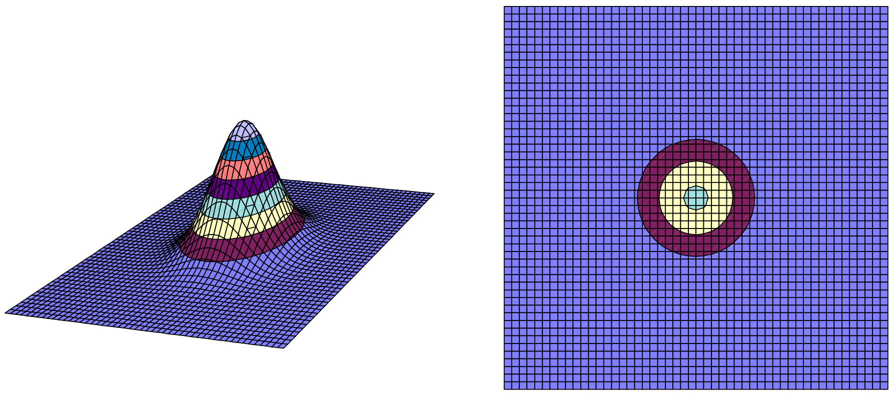

Figure 3-1 Probability Density Distribution for a Point Datum with Top View

3.3.2 To find the POC of a square circumscribed about this circle it is necessary to use the standard normal distribution table which can be found in most probability and statistics texts or in books of mathematical tables. Such a table is reproduced in Appendix A (Table A-1) of this paper. If the radius of the circle is one standard deviation (σ), then the sides of the circumscribed square are two standard deviations in length and go from negative one standard deviation to positive one standard deviation as shown in Figure 3-2. The probability that a point represented by the ordered pair (x,y) is in the square is the joint probability that x is between plus and minus one standard deviation **and** y is also between plus and minus one standard deviation. Since the distributions of x and y values are both assumed to be normal, these probabilities may be computed from standard normal distribution tables as follows. Most tables give the area under the standard normal function from negative infinity to the value used to enter the table. The standard normal curve is symmetric about zero, so the probability of a normally distributed variate being less than zero is 0.5 or 50%. The probability that it is between zero and one standard deviation is 0.84 - 0.5 = 0.34 or 34% and the probability that it is between plus and minus one standard deviation is exactly twice this value or 68%. Therefore, the probability that x lies between plus and minus one standard deviation is 68%, the probability that y lies between plus and minus one standard deviation is also 68% and the joint probability that both lie within these bounds at the same time is 0.68 x 0.68 = 0.47 or 47%. This makes sense since the square includes more area, and hence more of the distribution, than the circle.

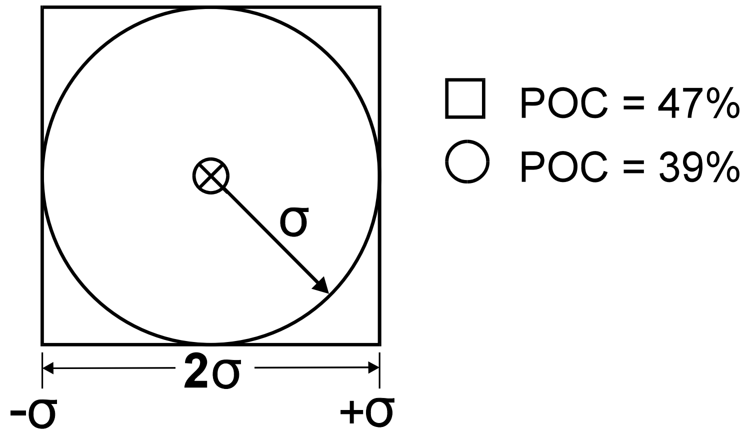

Figure 3-2

3.3.3 Although the standard deviation is commonly used as a reference value in probability and statistics, in navigation the common reference is *total probable error of position*, usually denoted by *E*. Position errors are commonly assumed to have a circular normal distribution. The total probable error of position is taken to be the radius of the circle which has a POC of 50%. To determine the radius of such a circle, it is necessary to solve equation [3-1] above for *R*. Doing this produces

[3-2] $$R = \sqrt{2\ln(1POC)}$$

where **ln** is the natural logarithm function. Substituting 0.5 (50%) for POC to find *E* gives

$$E = \sqrt{2\ln(0.5)} = 1.1774.$$

That is, a circle with a radius of about 1.18 standard deviations contains 50% of a circular normal distribution.

3.3.4 Using the same technique as in subparagraph 3.3.2 above, the POC of a square circumscribed about a circle of this size can be computed. The probability of *x* being between 0 and 1.1774 standard deviations is 0.8805 - 0.5 = 0.3805 so the probability that it lies between -1.1774 and +1.1774 is twice this value or 0.7610. The probability that both *x* and *y* lie within these limits is 0.761 x 0.761 = 0.5791 or about 58%. These computations may be written more compactly as

$$[2(0.88050.5000)]^2 = 0.5791$$

or 57.91%. Figure 3-3 illustrates this situation.

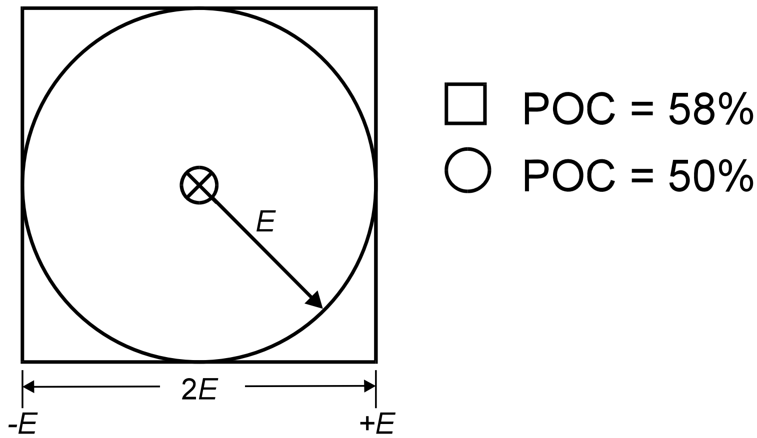

Figure 3-3

3.3.5 Using the standard normal distribution tables, it is possible to construct a *probability map* of a circular normal distribution. This is done by laying a grid over the distribution and computing the POC value for each cell in the grid. A probability map for a point datum is shown in Figure 3-4. In the last subparagraph, the computations for the POC of the central square in the grid were done. To illustrate how the values for the other grid cells were computed, the value for the shaded cell will be computed below. The probability that *x* lies between *E* and 3*E* is 0.9998 - 0.8805 = 0.1193. The probability that *y* lies between -*E* and +*E* is 0.7610. Therefore, the probability that the search object is in the shaded cell is

$$0.1193x0.7610 = 0.0908$$

or 9.08%.

1.42%	9.08%	1.42%
9.08%	57.91% (E)	9.08%
1.42%	9.08%	1.42%

Figure 3-4 Probability Map for a Point Datum

3.3.6 In the SSPM, it is recommended that the first search radius be 1.1 times the total probable error of position and that the first search area be a square circumscribed about a circle of that radius. The radius of such a circle is 1.1 x 1.1774 = 1.2952 standard deviations. The POC of the circumscribed square, using standard normal probability tables, is

$$[2x(0.90240.5000)]^2 = 0.6477$$

or about 65%. Figure 3-5 illustrates this situation.

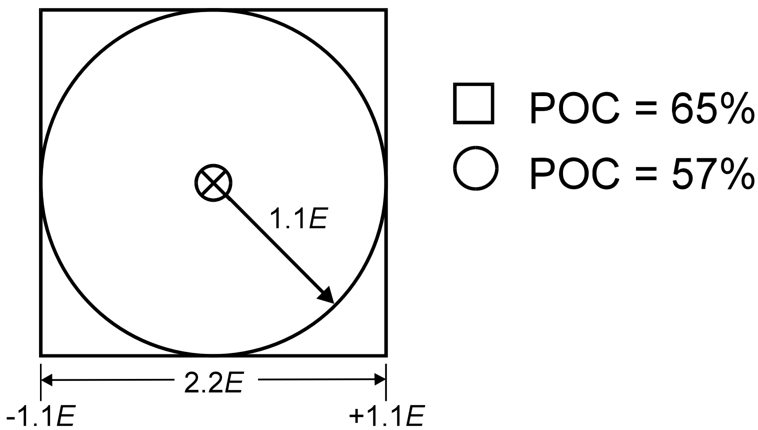

Figure 3-5

3.3.7 The SSPM assumes a POD of 79% on all searches. In Chapter 4, it will be shown how this value is computed based on a number of other assumptions. Allowing, for the moment, that this assumption is correct, the POS for the first search would be, using equation [2-3] from the last chapter,

$$POS = 0.6477x0.79 = 0.5117$$

or about 51%. Achieving a POS of at least 50% on the first search may have been one of the goals of the developers of the SSPM. This goal and the assumption of a 79% POD would have led the SSPM developers to the 1.1 first search “safety factor” as it is called in that method or “optimal search factor” as it is called in the modern, improved search planning method (ISPM). Based on the assumptions which produce the 79% POD value (discussed in Chapters 4 and 5), the resulting allocation of search effort is, in fact, very nearly optimal. It is unlikely that this is merely a coincidence.

3.4 Line Datums. The simplified method, in its original form, was based solely on the point datum. It was extended to include situations where a vessel or aircraft was known or suspected to have experienced a distress while traveling along a straight line connecting two points.

3.4.1 The extension of the SSPM to include line datums was done as follows. For the first search, the total probable error of each position was estimated, multiplied by 1.1, and circles of the corresponding radii were drawn at the end points of the line and "boxed in" as shown in Figure 3-6. Figure 3-7 shows a three-dimensional representation of the probability distribution implied by this technique. It was then assumed that searching the "boxed in" area with a 79% POD would produce the same results as searching the square recommended for point datums. It is relatively difficult to create the distribution shown in Figure 3-6 and determine the results of searching areas of different lengths and widths so it is unlikely that these assumptions were ever verified.

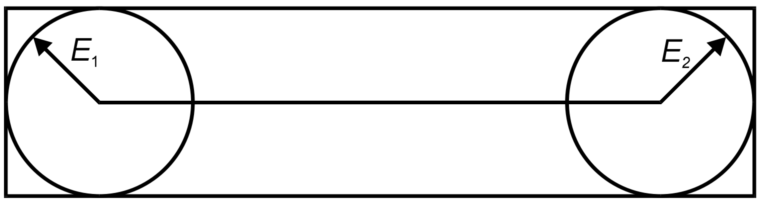

Figure 3-6

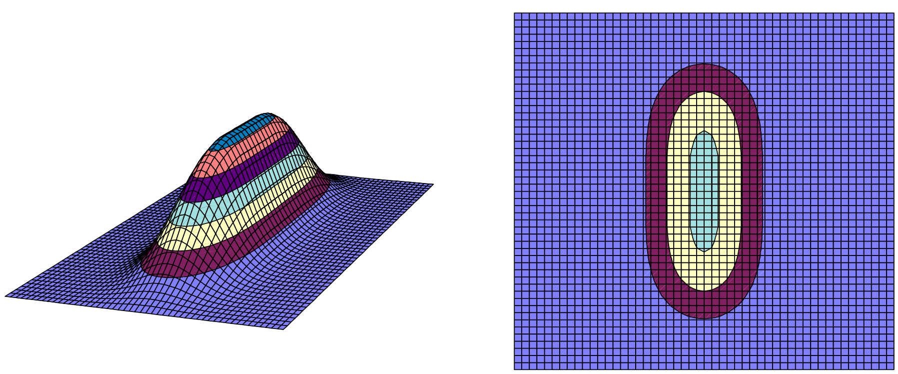

Figure 3-7 Probability Density Distribution for a Line Datum Connecting Two Point Datums with Top View

3.4.2 It is reasonably easy, however, to create and work with a simpler but similar distribution. If it is assumed that the search object's probable locations perpendicular to the datum line are described by a normal distribution centered on the datum line, and that its probable locations parallel to the line are uniformly distributed between the two end points (and zero beyond them), a distribution like that shown in Figure 3-8 is produced. The probability of the search object being in any strip parallel to the datum line can be easily computed from standard normal tables. For example, the probability of the search object being within one standard deviation either side of the datum line is

$$2(0.84130.5000) = 0.6826$$

or about 68%.

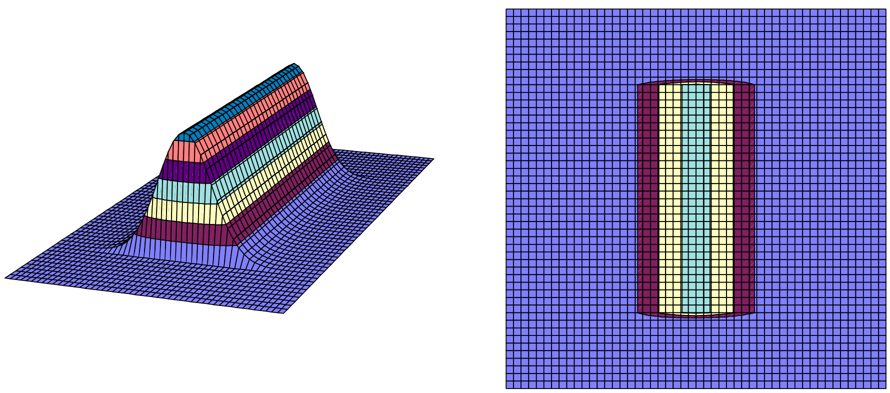

Figure 3-8 Probability Density Distribution for a Line Datum with Top View

3.4.3 Another interesting computation is that which produces the POC for a strip centered on the datum line which has a width equal to twice the total probable error of position, *E*, as shown in Figure 3-9. The POC for this rectangle is

$$2(0.88050.5000) = 0.7610$$

or about 76%.

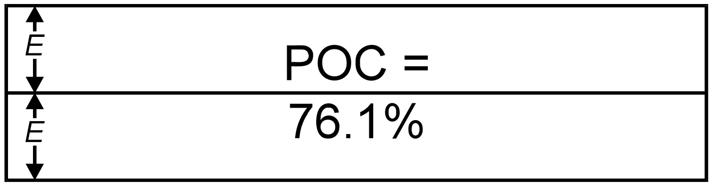

Figure 3-9

3.4.4 Figure 3-10 shows a probability map for a line datum. The POC values for strips parallel to the datum line are computed using the standard normal tables. These strips were then divided along their length and assigned a fraction of the strip's probability according to the width of the cell in proportion to the length of the strip. That is, if the cell's width is one-tenth the length of the strip, then the cell is assigned one-tenth of the strip's POC.

	0.2%	0.2%	0.2%	0.2%	0.2%	0.2%	0.2%	0.2%	0.2%	0.2%
	2.2%	2.2%	2.2%	2.2%	2.2%	2.2%	2.2%	2.2%	2.2%	2.2%
Datum Line →	5.2%	5.2%	5.2%	5.2%	5.2%	5.2%	5.2%	5.2%	5.2%	5.2%
	2.2%	2.2%	2.2%	2.2%	2.2%	2.2%	2.2%	2.2%	2.2%	2.2%
	0.2%	0.2%	0.2%	0.2%	0.2%	0.2%	0.2%	0.2%	0.2%	0.2%

Figure 3-10 Example of a Completed Probability Map for a Line Datum

3.5 Area Datums. Sometimes the only thing known about a SAR incident's location is that it occurred within some area with known boundaries. In this case, the probability of the search object being located in one place is the same as its probability of being located in any other place within the area. That is, the probability density is uniformly distributed over the area. This makes the POC of any subarea proportional to its size in relation to the size of the possibility area. If the possibility area is divided into five subareas of equal size, then each subarea would be assigned a POC of 20%. Figure 3-11 shows a three-dimensional representation of a uniform distribution. Figure 3-12 shows a probability map for an area datum.

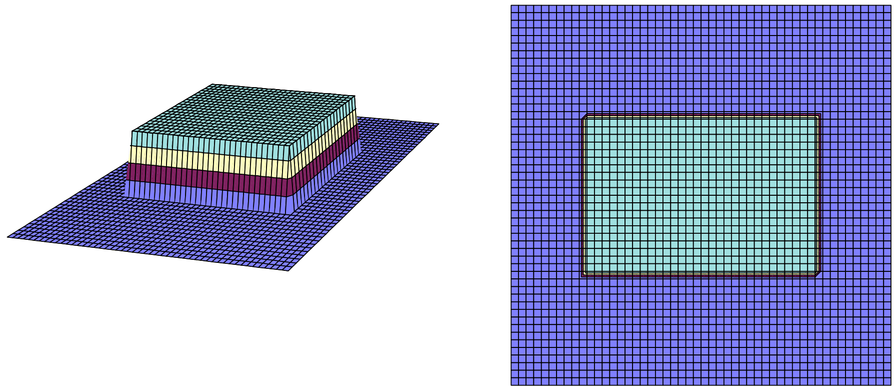

Figure 3-11 Uniform Probability Density Distribution with Top View

5%	5%	5%	5%	5%
5%	5%	5%	5%	5%
5%	5%	5%	5%	5%
5%	5%	5%	5%	5%

Figure 3-12

3.6 Generalized Datums. Sometimes none of the standard distributions seem to fit the circumstances of the case. In this situation, it is necessary to determine the possibility area, divide it into subareas suggested by the available information and finally assign estimated POC values based on a careful analysis of all available information. Figure 3-13 is a three-dimensional illustration how a generalized datum's probability distribution might look. Figure 3-14 shows an example of a generalized datum probability map.

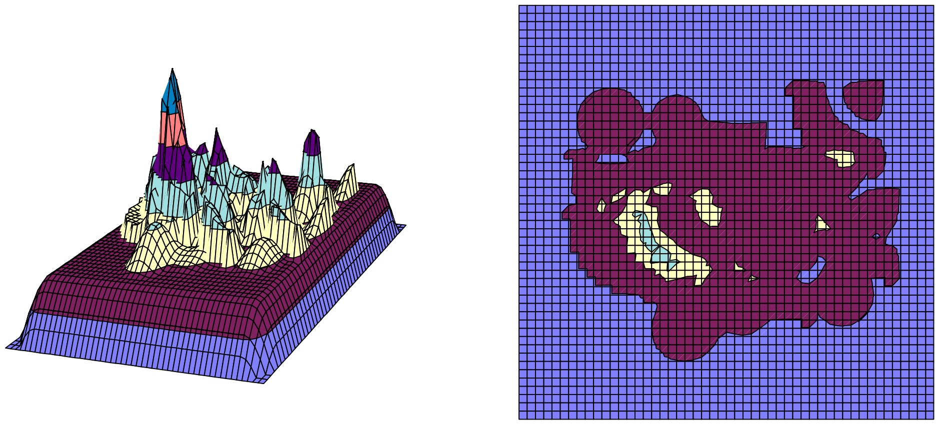

Figure 3-13 Generalized Probability Density Distribution with Top View

3%	4%	5%	6%	2%
2%	8%	15%	10%	3%
5%	10%	6%	5%	2%
4%	3%	4%	2%	1%

Figure 3-14

Chapter 4
Detection Models, Lateral Range Curves and Sweep Width

4.1 Introduction. This chapter discusses the characteristics of three specific detection models. They are *definite range*, *M-Beta*, and *inverse cube*. The first and simplest model is used to illustrate the concepts of *lateral range curves* and *sweep width*, applied to all three models. This chapter restricts its attention to the sensor models and the characteristics of their lateral range curves. In the next chapter, the performance of these models in search patterns consisting of equally spaced parallel tracks is examined.

4.2 Lateral Range Curves. Consider the situation where a sensor is moving through an area and the relative motion between the sensor and any search object in the area is a straight line. The lateral range, x, is the perpendicular distance from the sensor's relative track to the object, which is the same as the object's distance from the sensor at the closest point of approach (CPA). A lateral range curve describes the cumulative probability that an object will be detected as it passes once completely through the sensor's instantaneous detection envelope as a function of the object's lateral range x. Sensors may be classified by the nature or shape of their lateral range curves. Figure 4-1 shows a number of different lateral range curves. The significance of the dimension W on each of the curves in Figure 4-1 will become clear in the following paragraphs.

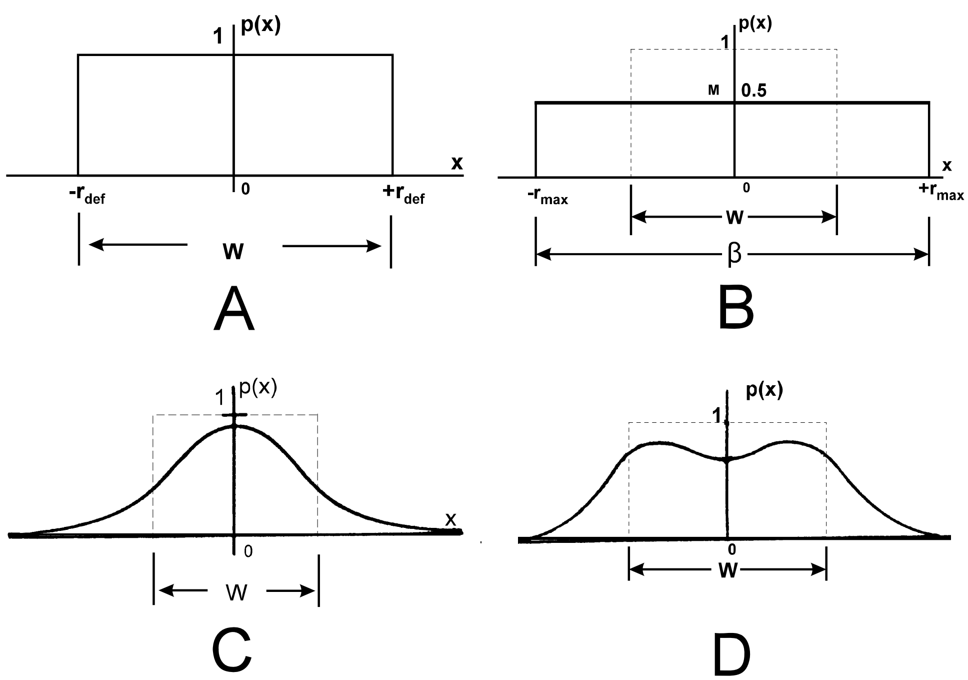

Figure 4-1

4.3 Sweep Width. Sweep width has the following interpretation. Assume the lateral range function is denoted by $p(x)$. If the sensor moves in a straight line through a population of objects uniformly distributed over the surface (N per unit area on the average) with either all at rest or all moving at the same vector velocity (so the relative motion between the sensor and all objects lies along straight parallel lines with a relative speed w), the average number N_S detected per unit time in any strip between the lateral ranges x_1 and x_2 depends on both the distance and the average value of $p(x)$ between x_1 and x_2 as well as the object density N and the relative velocity w. Specifically,

[4-1] $$N_s = (x_2 - x_1)\, Nwp_{avg}(x_1, x_2)$$

where the width of the strip (x_2 - x_1) times the object density N times the relative velocity w gives the number of objects in that strip passed per unit time and p_{avg} gives the fraction of those objects which are detected. However, in mathematical terms, the expression

$$(x_2 - x_1)\, p_{avg}(x_1, x_2)$$

is simply the *area under the lateral range curve between x_1 and x_2*. This means that the total number of objects N_T detected per unit time is Nw times the total area under the lateral range curve. That is,

$$N_T = Nw \int_{\infty}^{+\infty} p(x)dx\ .$$

4.3.1 The area W under a sensor's lateral range curve is called the *effective search (or sweep) width*. If the lateral range function is $p(x)$, then

[4-2] $$W = \int_{\infty}^{+\infty} p(x)dx$$

The *sweep width* equals the area under the *lateral range curve*.

4.3.2 Equation [4-2] provides one means of comparing sensors. Two sensors having the same sweep width are "equivalent" in the following restricted sense. Consider two identical, infinitely large, uniformly distributed sets of objects J_1 and J_2 and two distinct sensors K_1 and K_2 with different lateral range curves but equal sweep widths. Sensor K_1 will pass once through J_1 and sensor K_2 will pass once through J_2. Both will have the same relative velocity *w* with respect to the search objects. Regardless of any differences in the shapes of their lateral range curves, each sensor will detect the same number of objects, on average, per unit time. Although sweep width is a useful concept, this definition of "equivalence" between sensors has limited practical value and can be very misleading if applied to realistic, as opposed to purely theoretical, situations.

4.3.3 It could be said that a sensor with a larger sweep width is "better" than one with a smaller sweep width because it can detect more objects per unit time during a single sweep through an infinitely large area with a uniform distribution of search objects. In this sense, sweep width is at least a partial measure of the ease or difficulty of detection. However, most searches are performed by moving the sensors through limited areas along equally spaced parallel tracks where the spacing between tracks is of the same order of magnitude as the sweep width. The cumulative probability of detecting an object in such an area after all the parallel sweeps have been completed is highly dependent upon both the sweep width and the shape of the sensor's lateral range curve.

> The practical value of the sweep width concept lies in its usefulness for determining the amount of available *search effort* and the optimal *track spacing* in search patterns that employ parallel tracks. Both of these uses are explored in Chapters 5 and 6.

The remainder of this chapter examines three specific types of lateral range curves. In the next chapter, the relationship between lateral range curve shapes and corresponding PODs of parallel track search patterns is discussed using the lateral range curves described below.

4.4 The Definite Range Model of Detection. A definite range sensor is one that is 100% effective out to some definite lateral range from the sensor and completely ineffective beyond that range. That is, the *probability of detection* $p(x)$ for search objects within the definite range, r_{def}, is 100% while the POD for search objects further away is zero. Stated mathematically, the lateral range function is

$$p(x) = 1.0 \quad \text{if} \quad r_{def} \le x \le +r_{def}$$

[4-3]

$$p(x) = 0.0 \quad \text{if} \quad x < -r_{def} \quad \text{or} \quad x > +r_{def}$$

Figure 4-2 illustrates this characteristic where $p(x)$ is graphed as a function of lateral range x. Note that the definite range is also the maximum detection range, r_{max}, for this type of sensor.

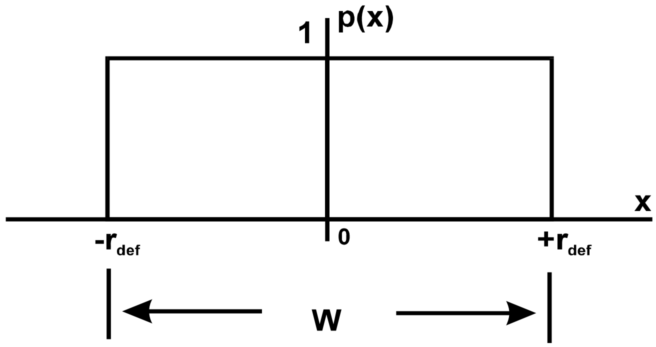

Figure 4-2

4.4.1 Computing the area under the lateral range curve shown in Figure 4-2 is a simple matter due to its rectangular shape. The height of the rectangle is 1.0 (100%) and the width is $2r_{def}$ so the sweep width is

$$W = 1.0x2\,r_{def} = 2\,r_{def}$$

Note that if the sensor is moved in a straight line, it will *sweep* an area whose width is $2r_{def}$ as shown in Figure 4-3. In this case, the width of the swept area is exactly equal to the sweep width, *W*. That is, for definite range sensors, the sweep width is literally the width of the swept area. From another point of view, it can be said that the sweep width of any sensor is the width a definite range sensor would have to sweep in order to detect the same number of objects per unit time in a uniform distribution of search objects.

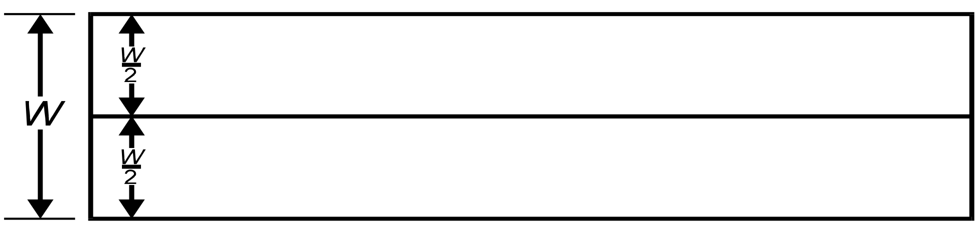

Figure 4-3

4.4.2 It is now possible to write the lateral range function in terms of the sweep width.

$$p(x) = 1.0 \quad \text{if} \quad -\frac{W}{2} \le x \le +\frac{W}{2}$$

[4-4]

$$p(x) = 0.0 \quad \text{if} \quad x < -\frac{W}{2} \quad \text{or} \quad x > +\frac{W}{2}$$

Expressing lateral range functions in this manner will be useful when comparing the performance of different sensors when used in parallel track search patterns.

4.5 M-Beta Detection Model. An M-Beta model is one that has a rectangular lateral range curve like the definite range model, except that the height of the rectangle is some value M between 0 and 1 and the width of the rectangle is some value β. The maximum detection range, $r_{\max}$, is then $\beta/2$. In fact, the definite range model is a special case of the M-Beta model where the uniform probability within the maximum detection range is 1.0 (100%) and $r_{\max} = r_{def}$. The M-Beta model is more general than the definite range model in that it can accommodate any probability (M) greater than 0 and less than or equal to 1 between $x = -r_{max}$ and $x = +r_{max}$. The general form of the M-Beta lateral range function is

$$p(x) = M \quad \text{if} \quad r_{\max} \le x \le +r_{\max}$$

[4-5]

$$p(x) = 0.0 \quad \text{if} \quad x < -r_{\max} \quad \text{or} \quad x > +r_{\max}$$

4.5.1 Figure 4-4 shows an M-Beta lateral range curve where M is only half as large as the definite range model (50% vs. 100%) but the maximum detection range is twice as large ($r_{max1} = 2r_{def}$). Hence, the height of the rectangular area under the lateral range curve is 0.5 (50%), the width is $4r_{def}$ and the sweep width is computed as

$$W = 0.5x2\,r_{\max 1} = 0.5x4\,r_{def} = 2\,r_{def}$$

which is exactly the same as the sweep width of the definite range sensor. This leads to the following observation.

The sweep width of any sensor is equal to twice the detection range of an equivalent definite range sensor, where “equivalent” means that each of the two sensors detects on average the same number of uniformly distributed search objects, each having relative velocity w, per unit time.

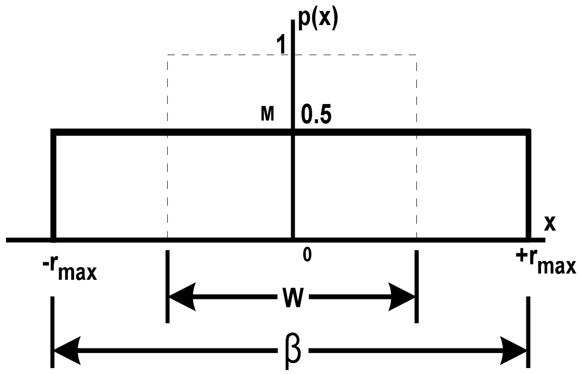

Figure 4-4

4.5.2 Note in the above example that both r_{max1} and W are equal to $2r_{def}$. This means that $r_{max1} = W$. Writing the lateral range function in terms of W for this sensor leads to

[4-6]

$$p(x) = 0.5 \quad \text{if} \quad W \le x \le +W$$

$$p(x) = 0.0 \quad \text{if} \quad x < -W \quad \text{or} \quad x > +W$$

4.5.3 Figure 4-5 shows another M-Beta lateral range curve. This time, M is 0.25 (25%) and the maximum detection range is four times that of the definite range model ($r_{max2} = 4r_{def}$). Again, because the maximum detection range has changed in inverse proportion to the change in M, the sweep width remains the same.

$$W = 0.25x2\, r_{\max 2} = 0.25x8\, r_{def} = 2\, r_{def}$$

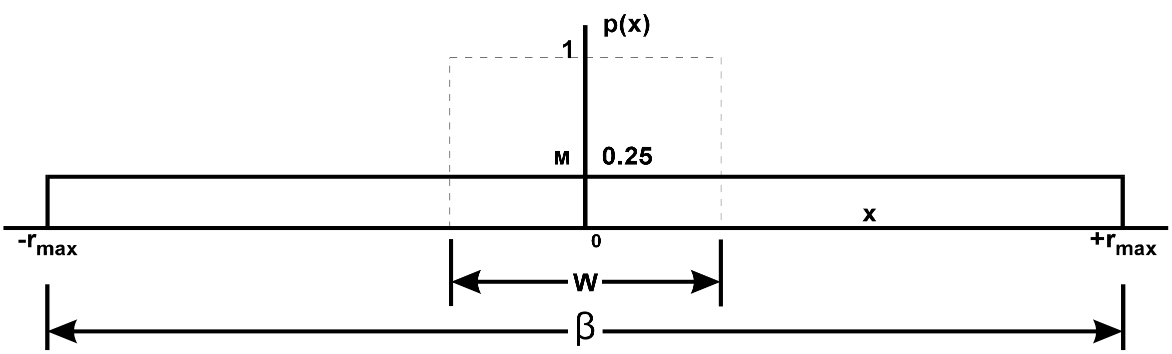

Figure 4-5

4.5.4 Now, $r_{max2} = 2W$ so the lateral range function in terms of W becomes

[4-7]

$$p(x) = 0.25 \quad \text{if} \quad 2W \leq x \leq +2W$$

$$p(x) = 0.0 \quad \text{if} \quad x < -2W \quad \text{or} \quad x > +2W$$

4.6 Inverse Cube Model. Both the definite range and M-Beta detection models are very simple and even useful for theorists. However, they are not realistic models of very many actual sensors, with the possible exception of certain electronic sensors under certain conditions. This was even more true when search theory was first being developed. Fifty years ago, electronic sensors were in their infancy and the exploration of their capabilities was just beginning. The human observer, searching visually, was still the primary sensor, just as in SAR today. It was clear even in the 1940s that neither of the above models was a good description of visual detection. A better description was sought and the inverse cube model was the result.

4.6.1 Even though humans had been using their eyes to search for objects for tens of thousands of years, by the 1940s when search theory was first being developed, little or no formal scientific research had been done on visual detection, at least not as it related to organized searching. As a result, the search theorists were left with no choice but to develop an empirical model. Koopman's research at that time was being done for the U.S. Navy and this is reflected in the assumptions that went into the model's creation. The basic assumptions were

- the search objects are warships underway;
- the search facility is a patrol aircraft flying at height *h* above the ocean;
- the mode of detection is an observer in a patrol aircraft sighting the warship's wake;
- the instantaneous (one glimpse) probability, γ, of sighting the vessel's wake is proportional to the solid angle subtended at the observer's eye by the wake's visible area.

4.6.2 The last assumption is illustrated in Figure 4-6 where it is assumed the wake is a rectangle of length *a* toward the observer and width *b* perpendicular to the direction of observation. The infinitesimal solid angle is the product of the angle α subtended by *a*, and the angle β subtended by *b*. The radian measure of α is c/s. By similar triangles, $c/a = h/s$ and hence $\alpha = ah/s^2$. The radian measure of β is b/s. Hence, the solid angle $\alpha\beta = abh/s^3$ = the area, *A,* of the rectangle times h/s^3 or Ah/s^3. The actual area *A* of the search object's wake may not be rectangular in shape but can be regarded as made up of a large number of rectangles like the above, the solid angle being the sum of the corresponding solid angles. Hence, when the dimensions of *A* are small in comparison with *h*, *r*, and *s*, we have the formula

[4-8] $$\frac{Ah}{s^3} = \frac{Ah}{(h^2 + r^2)^{\frac{3}{2}}}.$$

Since γ is assumed to be proportional to the solid angle,

[4-9] $$\gamma = \frac{kh}{s^3} = \frac{kh}{(h^2 + r^2)^{\frac{3}{2}}},$$

where the constant of proportionality, *k*, depends on all the factors we regard as fixed without introducing explicity, such as contrast of wake against ocean, observer's ability, meteorological conditions, etc.; and of course *k* contains *A* as a factor. Dimensionally, $k = [L^2T^{-1}]$, or area per unit time. In the majority of cases, *r* is much larger than *h*, and equation [4-9] can be replaced by the satisfactory approximation

[4-10] $$\gamma = \frac{kh}{r^3}.$$

It is from this formula that the *inverse cube* model of visual detection gets its name. Translated into words, the formula states that the instantaneous probability γ that the search object will be detected is inversely proportional to the cube of the range *r* from the observer to the object

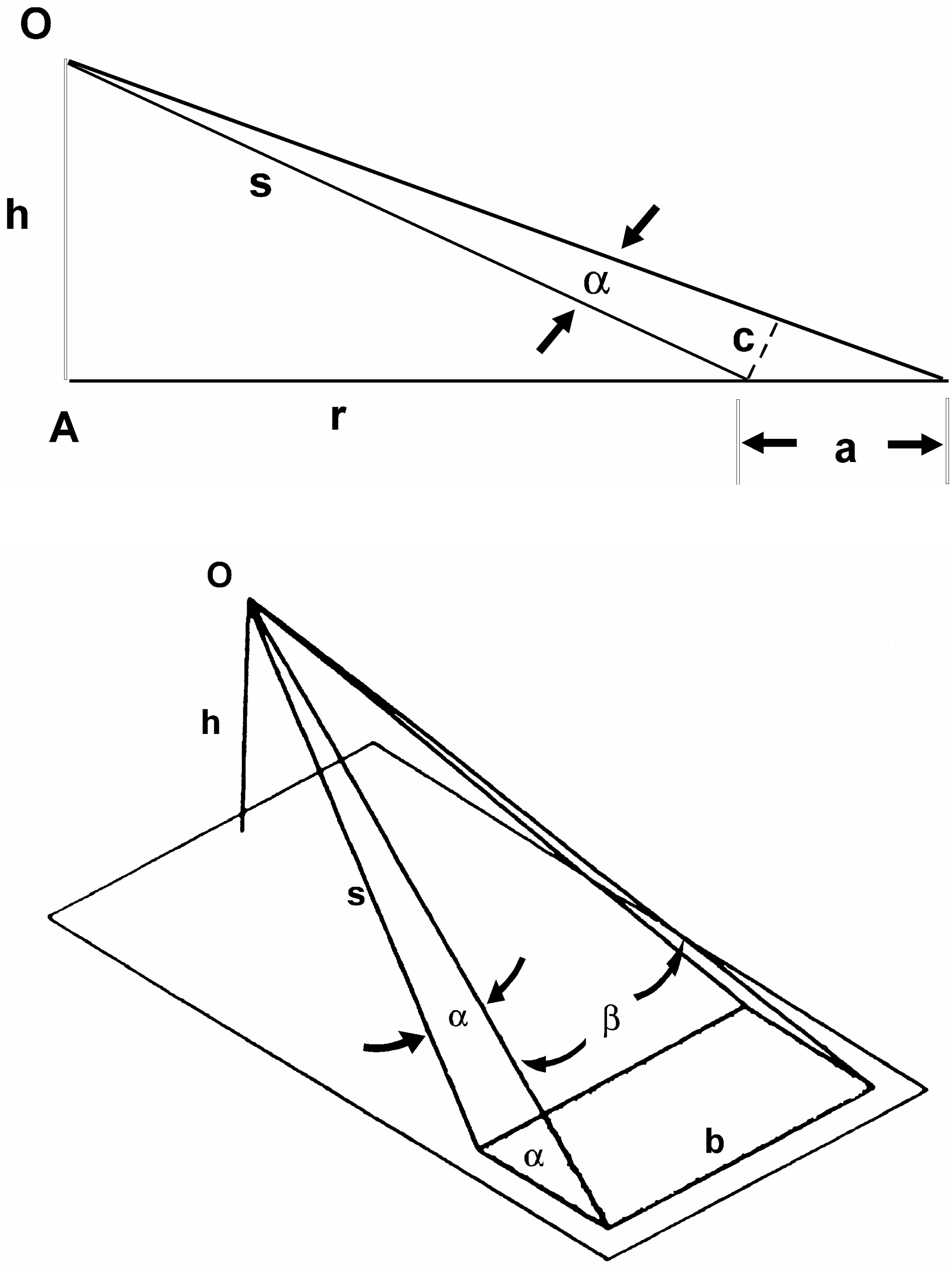

Figure 4-6

4.6.3 In the previous examples (definite range and M-Beta), a lateral range curve was assumed outright and without regard to the instantaneous detection characteristics of the sensor itself. However, with the inverse cube sensor, a function that describes the instantaneous detection probability has been developed. It is now necessary to determine the lateral range curve which will result when a sensor with this instantaneous detection function is moved along a straight line relative to the search objects. Finding this lateral range curve requires some relatively involved mathematics which may be found in Koopman [1]. Here the result, based on [4-10] above, will be stated without a formal derivation.

[4-11] $$p(x) = 1\ e^{\frac{2kh}{wx^2}}$$

Integrating [4-11] to find the sweep width (which requires a change of variable and integration by parts, steps which are not shown),

[4-12] $$W = \int_{\infty}^{+\infty} 1\ e^{\frac{2kh}{wx^2}} dx = 2\sqrt{\frac{2\pi kh}{w}}\ .$$

It is now possible to write [4-11] as a function of *W*.

[4-13] $$p(x) = 1\ e^{\frac{W^2}{4\pi x^2}}$$

4.6.4 The graph of [4-13] appears in Figure 4-7. Note that the "tails" of this bell-shaped curve approach the *x* axis from above as an asymptote. This means that although $p(x)$ becomes vanishingly small as *x* increases without bound, it never quite becomes exactly zero. Therefore, the maximum detection range, in theory, is infinite. Also note that as the lateral range *x* approaches zero, $p(x)$ approaches 1.0 (100%).

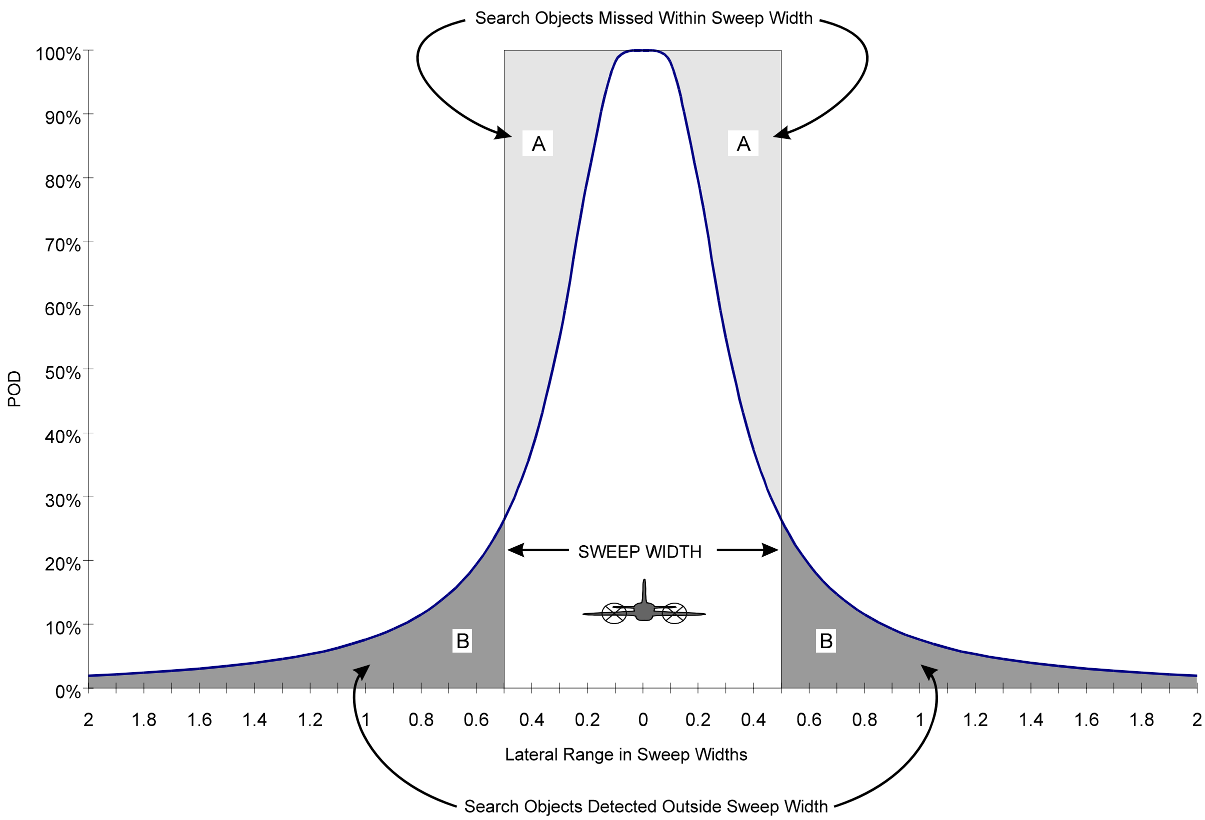

Figure 4-7

4.7 Importance of Sweep Width. In paragraphs 4.4-4.6 above, three different sensor types and their lateral range functions were examined. It is now possible to note some common characteristics in how each relates to its associated sweep width.

4.7.1 If the lateral range function and sweep width are known for a particular sensor, search object, and set of environmental conditions, then the probability of detecting the search object at a particular lateral range may be computed. This in turn will allow search planners, as shown in the next two chapters, to determine the best *track spacing* to use in a parallel track search pattern.

4.7.2 Another item worthy of note is that the formula for the sweep width W in [4-12] involves not only the height of the observer h and the relative speed w, but the constant of proportionality k which in turn involves a great many other factors.

Sweep width depends on all the characteristics of the *sensor*, *search object*, and *environment* which affect detection.

A very small sampling of the factors affecting sweep width for visual search includes

- for the observer: training, visual acuity, fatigue, distractions of other duties, etc.;
- for the search object: size, color, contrast with surroundings, use of visual signaling devices (e.g. lights, flares, smoke, dye), etc.;
- for the environment: visibility, lighting conditions, sea state or type of terrain, etc.

There are also many interdependencies among these factors. For example, rough seas will increase fatigue among searchers aboard a vessel as well as make it more difficult for them to see small objects on the ocean's surface. In general, larger objects have larger sweep widths, sweep widths are higher for the same object when the air is clear than when it is hazy, higher for well-rested observers than for tired ones, higher in smooth terrain with little vegetation than in rough, heavily forested terrain, etc.

4.8 Assessing the Inverse Cube Model. Although the assumptions on which the inverse cube model is based are clearly not valid in the vast majority of SAR searches, this is the model on which all SAR search planning has been based for the last 50 years or so.

4.8.1 The main advantage of the inverse cube model is that unlike the definite range, M-Beta, or models developed for electronic sensors, it is based on a physical model, however crude or inaccurate, of visual detection under operational conditions. Unfortunately, few if any attempts have been made to determine the true instantaneous visual detection function for any search objects, large or small, under any environmental conditions. This is not too surprising since so many factors can affect visual detection in so many ways that building a really accurate model of the instantaneous detection function may not be practical.

4.8.2 The amount of data required to infer a sweep width or lateral range curve directly is orders of magnitude less than that required to develop an instantaneous detection model. As a result, some attempts have been made to obtain better estimates of sweep width and the factors affecting it. This has been done by recording the ranges at which various search objects are detected under various conditions. The most notable of these attempts have been experiments conducted over the past two decades by the U. S. Coast Guard Research and Development Center. The data that have been collected have been used to develop gross estimates of lateral range curves and corresponding sweep width values. However, the use of these values in practice is still based on the *assumption* that the inverse cube model is valid in all situations. Under ideal conditions involving parallel sweep searches for typical SAR search objects of moderate to large size, the inverse cube model often predicts the POD surprisingly well. However, for poor conditions or small search objects, the inverse cube model is a poor predictor of POD, being generally too optimistic. Fortunately, there is a practical way to deal with this problem using the theory of "random search" and it will be explored in the next two chapters.

sChapter 5
Searching Areas

5.1 Introduction. As a practical matter, searching is done in areas of limited size. It is common to speak of expending “search effort” and “covering” an area. These terms have already been used in previous chapters in a rather vague, generic way. However, the terms *search effort* and *coverage* have mathematically precise definitions in search theory. Both definitions involve *sweep width* either directly or indirectly and this chapter will begin with these topics. Next, there is a discussion of *random search* which also has a mathematically precise formulation. Random search is examined for three reasons. First, it has value as a benchmark or baseline against which organized search techniques may be compared and judged. Second, it will be shown that certain types of lateral range curves produce nearly the same results as would be obtained by random search, even when used in a highly organized, orderly fashion. Third, the random search curve may actually be a reasonable estimator of POD when all the unknowns (randomness) involved in actual search operations are considered. The remainder of the chapter will then be devoted to the study of how the sensors whose lateral range curves were studied in the previous chapter perform when used with parallel track search patterns to cover limited areas.

5.2 Search Effort. The word “effort” has many meanings and just as many methods of measurement. Ask a pilot how much effort is required to do a certain search and the answer is likely to be given in terms of flight hours required or number of trackline miles that must be flown to complete the assigned search pattern. However, this will not be a very good measure of how much searching will be done. If two different searches are the same in all respects except that the sweep width in one is twice what it was in the other, then in some sense twice as much searching was done in the search with the larger sweep width as compared to the search with the smaller sweep width. In search theory search effort is defined as follows.

> *Search Effort* (*Z*) is equal to sweep width times trackline miles in the search area, or alternatively, sweep width times search speed times hours spent in the search area.

That is, mathematically,

[5-1] $$Z = WxL$$

where *Z* is the search effort, *W* is the sweep width and *L* is the trackline length, or distance traveled, by the sensor within the search area. Equivalently,

[5-2] $$Z = WxVxT$$

where *Z* is the search effort, *W* is the sweep width, *V* is the search speed, and *T* is the amount of time the sensor spent in the search area. The units of measure for search effort are those of area (e.g. square nautical miles). Loosely speaking, search effort may be thought of as the amount of area which can be effectively swept.

5.3 Coverage. While search effort may define how much area can be swept, it doesn't indicate whether the search facility spent its time in a small area doing a highly concentrated search or in a large area doing a more cursory search. In the first situation, it would be natural to say the "coverage" of the area was "high" and in the second case it would be just as natural to say it was "low." A mathematically precise way to compare the two "coverages" is to compare the amount of *search effort* (as defined above) expended in each area to the size of that area. The most appropriate way to compare search effort with search area is to take the ratio of these two quantities. This produces the following definition for coverage.

Coverage is the ratio of the search effort to the area searched.

Expressed mathematically, this definition becomes

[5-3] $$C = \frac{Z}{A}$$

where *C* is the coverage (sometimes called *coverage factor*), *Z* is the search effort as defined in paragraph 5.2 and equation [5-1] or [5-2] above, and *A* is the area that was covered by the search effort. The quantities *Z* and *A* must be expressed in the same units of measure (e.g., square nautical miles), so *C* is a unitless quantity.

5.4 Random Search. In Chapter 1 it was observed that common experience and intuition indicate a search which follows some organized plan or pattern has a greater chance of succeeding than just looking around at random. This is because with an organized approach, an approximately uniform coverage of the area being searched can be guaranteed while with random searching it is likely that some spots will be covered at least twice and others won't be covered at all. As a result, some effort will be wasted. However, randomness in searching need not always be the fault of the searcher. Recall that in Chapter

4, the concepts of lateral range curves and sweep widths were based on the assumption that the *relative motion* between the observer and the search object was a straight line. If the search object is in motion, then any randomness in that motion will introduce randomness in its track relative to an observer. This situation is common in SAR where there is often a great deal of uncertainty about survivor movements while searches are in progress following a distress incident. For example, survivors adrift on the ocean move with the winds and currents. While some limited success has been achieved in predicting average drift motion over moderately long periods (from one to a few days), the motion of a drifting object on shorter time scales can be, and usually is, quite convoluted and "random." This is not surprising since a large random element would be expected in the motion of a small solid object suspended at the turbulent interface between two huge fluid masses (the ocean and the atmosphere). In any event, there are also myriad additional sources of randomness in SAR operations including navigational error, wind shifts, weather changes, and various distractions and unexpected events during the search. The frequently quoted line, "The best laid plans of mice and men oft go astray." is a poet's accurate observation that almost all humanly planned endeavors are subject to unavoidable random factors and are rarely completed exactly as intended.

5.4.1 To derive a formula for the probability of detecting a search object in an area under conditions of random search, consider the following three assumptions:

1. The search object is in an area of size A, is motionless, and has a location probability density that is uniformly distributed throughout the area.
2. The observer's path in the area is random in the sense that it can be thought of as having its different (not too near) portions placed independently of one another in the area.
3. On any portion of the path that is small relative to the total length of the path but decidedly larger than the range of possible detection, the observer always detects the search object if it is within the lateral range $W/2$ on either side of the path and never detects search objects beyond that range.

Now suppose the observer's path of length L is divided into n equal portions each of length L/n. If n is large enough that most of the pieces are randomly related to any particular one, the chance of failing to detect during the whole path is the product of the chances that detection will fail during motion along each piece. If, further, L/n is such that most of the pieces of this length are practically straight and considerably longer than the range of detection, then because the sensor is of the definite range type, the chance of detection is equal to the probability of the object being in the area swept. The area swept is simply WL/n and the probability of the object being in the swept area is WL/nA.

The chances of the observer **not** detecting the search object (i.e. the *PFail)* is then 1- *WL*/*nA*. The product of all these *PFail* values is the probability that no detection will take place anytime during the entire search. Therefore, the probability of detection (POD) for the search is given by

[5-4] $$POD = 1\left(1\frac{WL}{nA}\right)^{n},$$

or, for large *n*,

[5-5] $$POD = 1\,e^{\frac{WL}{A}}.$$

From equations [5-1] and [5-3] it is seen that the expression for coverage may be written as

[5-6] $$C = \frac{WL}{A}.$$

Hence, [5-5] may be rewritten as

[5-7] $$POD = 1\,e^{C}$$

which is the random search formula in its simplest form. The graph of this function is shown in Figure 5-1.

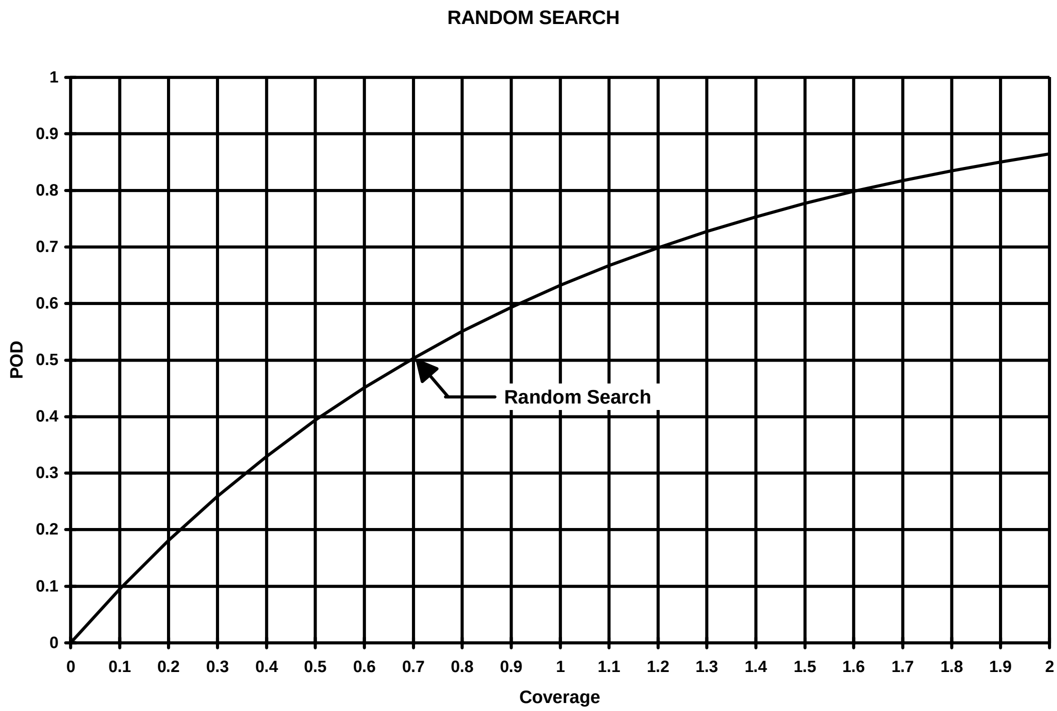
RANDOM SEARCH
POD
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
Random Search
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1 1.1 1.2 1.3 1.4 1.5 1.6 1.7 1.8 1.9 2
Coverage

Figure 5-1

5.4.2 The random search formula has some interesting properties. However, before examining these, it is important to observe a subtle difference between this chapter and the last. In Chapter 4, the main topics were lateral range functions which describe the probability of detecting a search object based on its lateral range from the observer's track. Equation 5-5 describes the probability of detection in an *area* and is **not** the formula for any lateral range function even though it's appearance is superficially similar to that of the inverse cube model. In this chapter, as in Chapter 2, probability of detection when used without qualification refers to POD — the probability of detecting a search object if it is in the area being searched.

5.4.3 One of the random search formula's more interesting properties is that it is solely dependent on the coverage C. The dependence of C on the lateral range curve is in turn limited to W which only depends on the area under the lateral range curve and no other characteristics. Thus, even though a definite range model was used to derive the random search formula, it is valid for virtually any reasonable detection model and certainly for any where $p(x)$ is maximum at $x = 0$ and decreases monotonically as $|x|$ increases.

5.5 Parallel Sweep Searches. The most common method of covering an area with search effort is to do a series of parallel sweeps. Figure 5-2 shows a typical parallel sweep (PS) search pattern. The dashed rectangle represents the search area. There are many practical benefits to conducting search operations in such an orderly fashion. One of those benefits is the ability to achieve a higher POD, and consequently a higher POS, in an area than would be achieved by random, disorganized searching. To see when and why this is true, the paragraphs that follow will compute parallel sweep POD values for several coverages for each of the lateral range curves examined in Chapter 4. The results will then be graphed and compared to the random search curve

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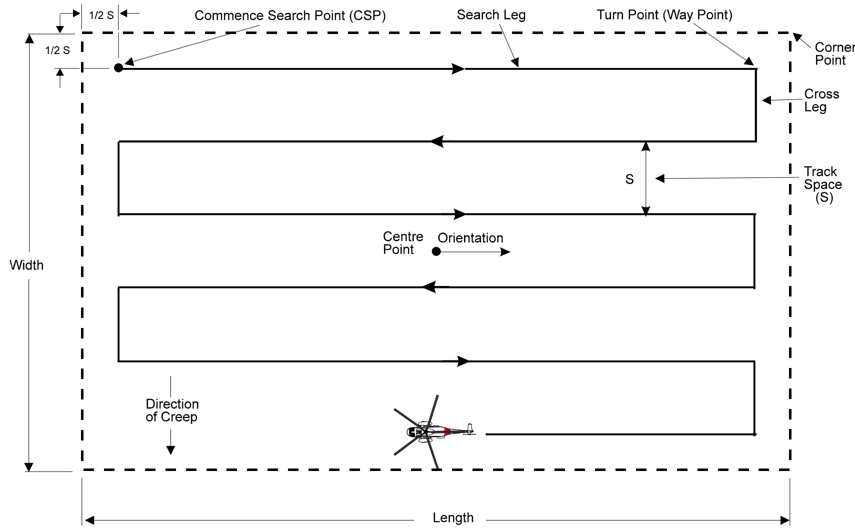

Figure 5-2 Parallel Sweep Search

5.6 Definite Range Model. Evaluation of the performance of this and the other detection models in parallel sweep searches will be done as follows. Several copies of the model's lateral range curve will be placed next to each other so that the distance from the center of one to the center of the next is equal to the track spacing for which the POD is being evaluated. In some cases, the lateral range curve from one track may overlap those from other tracks. In these cases, each detection opportunity will be treated as an independent event (like each toss in a series of coin tosses) and the POD in the space between the centermost two tracks computed accordingly. PODs for each of several track spacings (coverages) will be computed and then fitted with a curve which will be compared to the random search POD curve and the POD curves derived from the other lateral range curves.

5.61 As seen in the last chapter, the lateral range curve for the definite range model is a simple rectangle of height 1.0 and width *W* centered on the sensors track. Placing two tracks next to one another with a distance between them of 2*W* produces the condition shown in Figure 5-3. Note that at this track spacing, one half of the space between the tracks is covered and any object in this space that is also within *W*/2 of one of the two tracks will be detected. Objects in the central region that has a width of *W* will be missed. Both the *coverage* of the area between the tracks and the POD are 0.5. If the tracks are moved closer together so that the distance between them is only *W*, the lateral range curves from the two tracks abut one another as shown in Figure 5-4. In this case, both the coverage and the POD are 1.0. Pushing the tracks even closer together will produce no benefit in this case since the POD is already at its maximum value of 100%. The graph of the definite range POD function is plotted in Figure 5-5 along with the random search POD function for comparison.

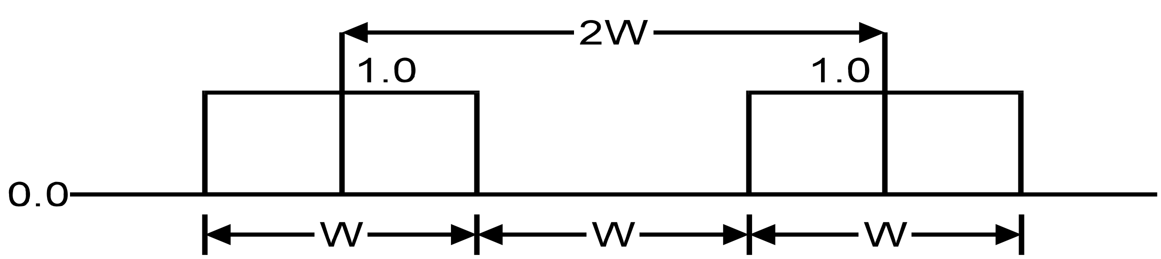

Figure 5-3

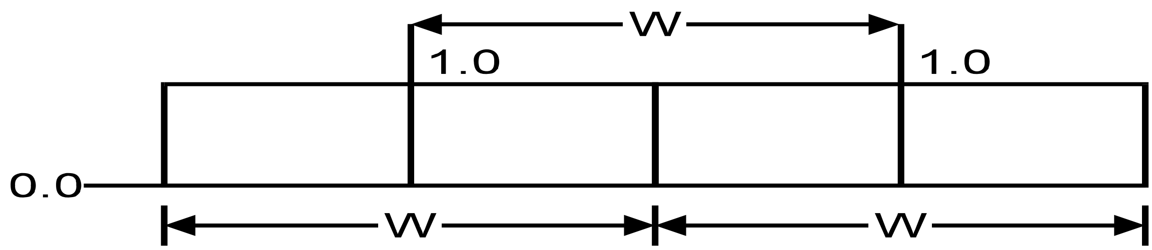

Figure 5-4

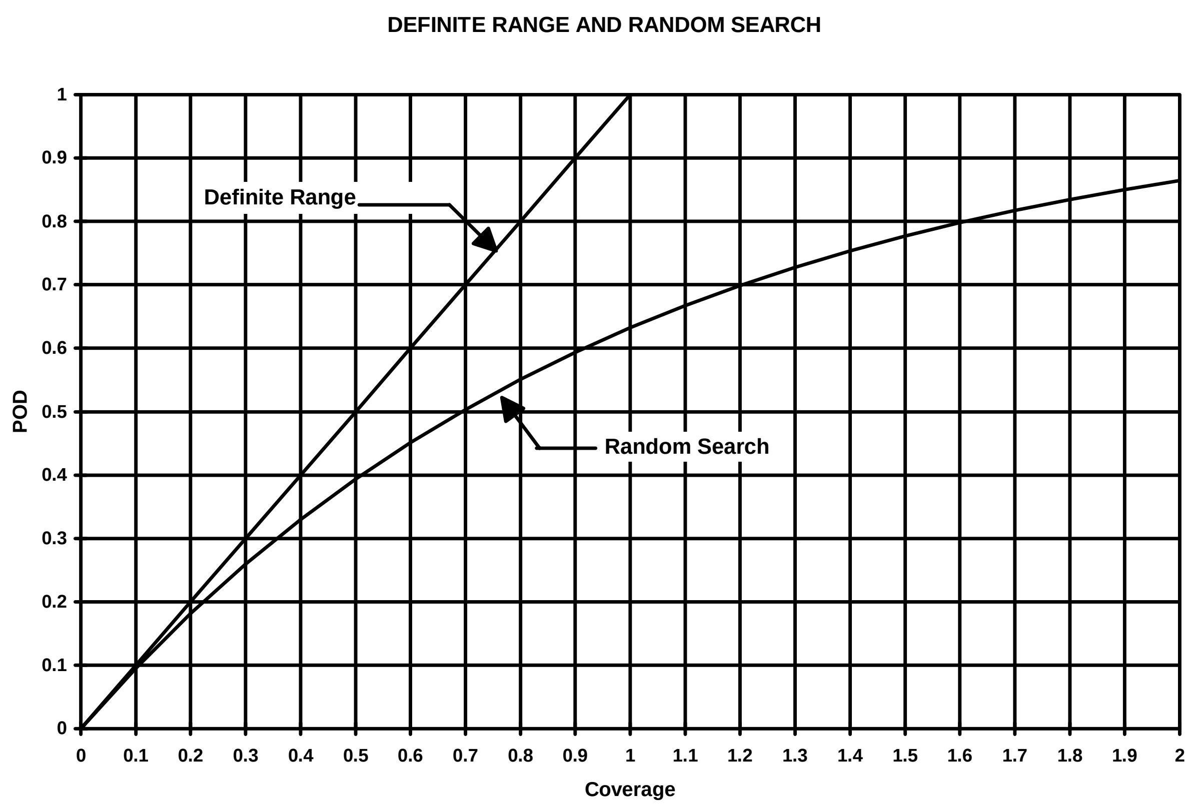

Figure 5-5

5.6.2 The definite range POD curve has several interesting properties. The first is that for coverages between 0 and 1.0 it is linear with a slope of 1.0 from the origin to (1,1). Second, because the definite range model is in some sense a "perfect" sensor, the definite range POD curve forms an upper bound for what can be expected of sensors employed in parallel sweep searches. Recall that the random search POD curve forms a lower bound on what should be expected of organized search efforts. It is anticipated, therefore, that the POD curves associated with all other models will fall between these two curves when used in parallel sweep search patterns.

5.6.3 Another observation may be made. For the definite range model, the ratio of the sweep width to the track spacing was exactly equal to the coverage. This provides a convenient shortcut method of computing the coverage for parallel sweep searches, namely

[5-8]

$$C = \frac{W}{S}$$

where *C* is the coverage, *W* is the sweep width, and *S* is the track spacing. The equivalence of this formula to [5-6] is easily shown and therefore it may be applied to all parallel sweep searches, regardless of the lateral range function in effect.

5.7 <u>M-Beta Model</u>. In Chapter 4, two M-Beta lateral range curves were described. Some representative POD values for different coverages will now be computed for each of these.

5.7.1 Consider the M-Beta lateral range function from 4.5.2 where

[5-9]

$$p(x) = 0.5 \quad \text{if} \quad W \le x \le +W$$

$$p(x) = 0.0 \quad \text{if} \quad x < -W \quad \text{or} \quad x > +W.$$

Figure 5-6 depicts the lateral range curves for two adjacent tracks which are spaced a distance of 2*W* apart. Note that the lateral range curves just touch one another which means all of the space between the tracks is being searched with a POD of 0.5. The coverage is also 0.5 and so, up to this point, the performance of this sensor is just as good as that of a definite range sensor. However, when the track spacing is decreased to *W*, making the coverage 1.0, the lateral range curves from the two adjacent tracks overlap and both completely cover the space between the two tracks. The cumulative POD in this space is

$$POD = 1(10.5)(10.5) = 0.75 \,,$$

which is well below the value for the definite range model (1.0) but still above the random search value (0.63) for this coverage. Pushing the tracks even closer together so that the spacing is only *W*/2 and the coverage is 2.0 causes the lateral range functions of four tracks to overlap and completely cover the spaces between tracks. For this situation, the POD is computed as

$$POD = 1(10.5)^4 = 0.9375 \,.$$

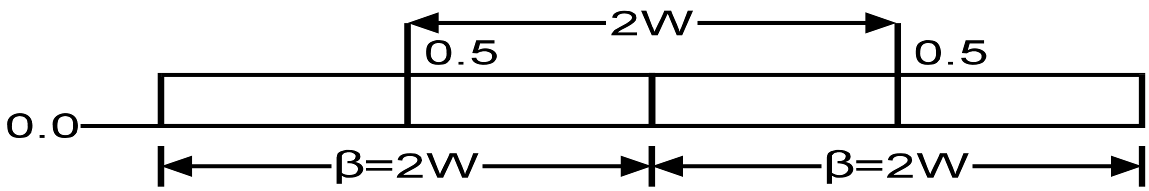

Figure 5-6

5.7.2 The other M-Beta model examined had a lateral range function given by

[5-10]

$$p(x) = 0.25 \quad \text{if} \quad 2W \le x \le +2W$$

$$p(x) = 0.0 \quad \text{if} \quad x <- 2W \quad \text{or} \quad x > +2W$$

In this case, the M-Beta model has the same POD values as the definite range model up through a coverage of 0.25. When the coverage is 0.5, lateral range curves from two adjacent tracks overlap, giving a POD of

$$POD = 1(10.25)^2 = 0.4375 \,,$$

which is closer to the random search curve (POD = 0.3935) than it is to the other two models (POD = 0.5). For a coverage of 1.0,

$$POD = 1(10.25)^4 = 0.6836 \,,$$

which is again closer to the random search curve (POD = 0.6321) than it is to the definite range (POD = 1.0) or the previous M-Beta (POD = 0.75) model. Finally, for a coverage of 2.0,

$$POD = 1(10.25)^8 = 0.8999 \text{ ,}$$

as compared to a definite range POD of 1.0, a POD of 0.9375 for the first M-Beta model and a random search POD of 0.8647. The POD graphs corresponding to these two variations on the M-Beta model are shown in Figure 5-7.

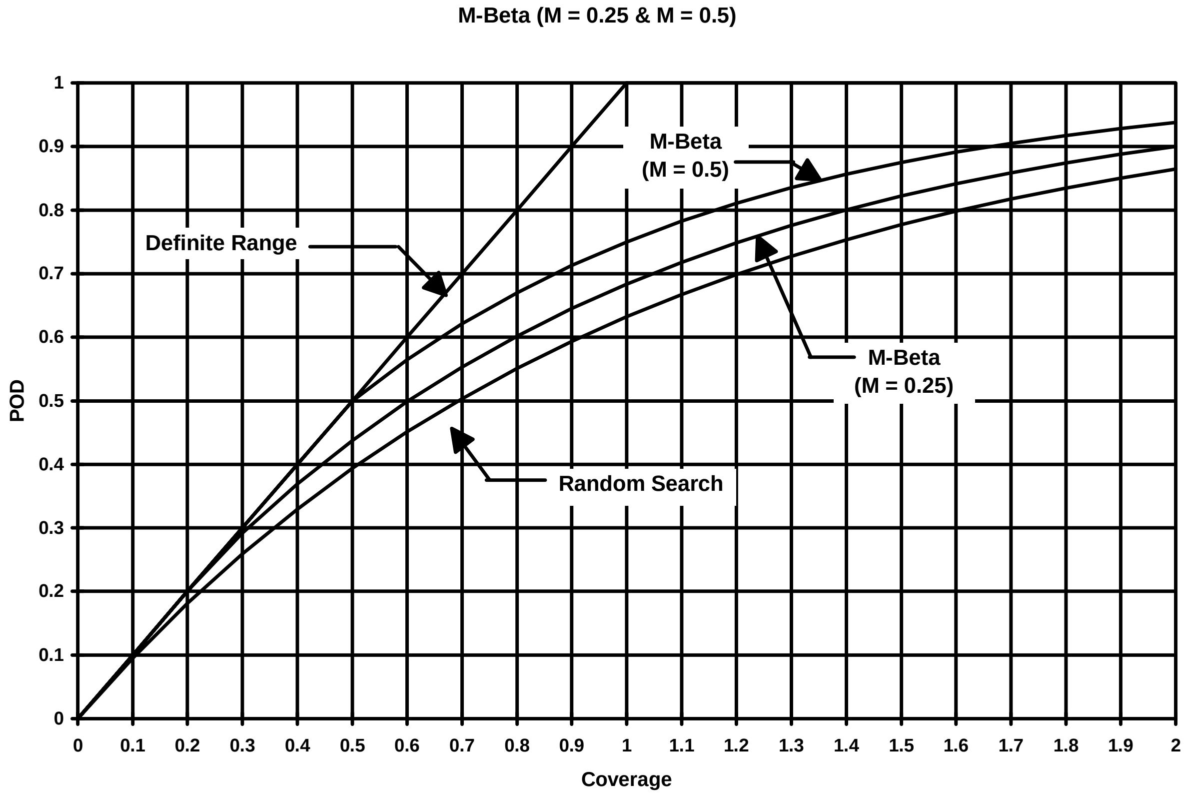

Figure 5-7

5.7.3 From the above examples, it is easy to see that the general formula for computing parallel sweep POD values for the M-Beta model is

[5-11]

$$POD = C \quad \text{if} \quad C < M \text{ ,}$$

$$POD = 1(1M)^{\frac{C}{M}} \quad \text{if} \quad C \geq M$$

It is also clear from the graphs of the examples that as *M* decreases in value, the corresponding POD function for parallel sweep searches approaches the random search curve. To see why this is true mathematically, it is only necessary to compare equations [5-4] and [5-11] for $C \geq M$. Note that the two expressions to the right of the equals sign have similar forms. In fact, doubling the value of *n* in [5-4] doubles the exponent while halving the value of *WL*/*nA* which occupies the same position as *M* in [5-11] when $C \geq M$. Similarly, halving the value of *M* in [5-11] has exactly the same effects. Therefore, as *M* becomes small, the POD function for M-Beta sensors performing parallel path searches approaches

$$POD = 1 e^{C} ,$$

which is the random search function. In geometric terms, as the lateral range function becomes lower, flatter, and more spread out, the more closely its parallel sweep POD function will approach that of random search. Intuitively this also makes sense because the accurate spacing of low, flat and very wide lateral range curves in relation to one another should not provide nearly as much benefit over random search as it does for high, peaked, and very narrow lateral range curves. This is an important observation which will be useful later in this chapter when dealing with how POD is affected by random influences such as navigational errors during the performance of parallel sweep searches.

5.8 Inverse Cube Model. Computing the POD in the space between the tracks in a parallel sweep search is conceptually the same for the inverse cube model as it was for the M-Beta model above. The differences lie in the difficulty of the computations (because *p*(*x*) now varies continuously with the lateral range *x* instead of being either a constant value or zero) and the infinite (in theory) maximum detection range of the inverse cube model. The latter condition requires the assumption that there are infinitely many equally spaced parallel tracks being considered (again, in theory). As with the derivation of the inverse cube lateral range function in the previous chapter, the reader is referred to Koopman [1] for the derivation of the inverse cube POD function for parallel sweep searches. Only the end result is provided here.

5.8.1 For an inverse cube sensor, the POD for a parallel sweep search is given by

[5-12] $$POD = erf\left(\frac{\sqrt{\pi}}{2}\frac{W}{S}\right)$$

where *erf* is the well-known error function. Equation [5-12] may be rewritten slightly to give POD in terms of coverage *C* as follows.

[5-13] $$POD = erf\left(\frac{\sqrt{\pi}}{2}C\right)$$

5.8.2 Like the standard normal probability density function encountered in Chapter 2, evaluation of the error function requires the use of tables. Table A-2 of Appendix A contains tabulated values for *erf*(*x*). For example, if *C* is 1.0, then

$$POD = erf\left(\frac{\sqrt{\pi}}{2} x1.0\right) = erf(0.8862).$$

Interpolating between the values for *erf*(0.88) = 0.7867 and *erf*(0.89) = 0.7918, the value 0.7899 or about 79% is computed. In other words, an inverse cube sensor, when used in a parallel sweep search at a coverage of 1.0 will produce a POD of 79%. The graph of [5-13] is shown as the centermost curve in Figure 5-8. This is the same well-known POD curve that has appeared in search and rescue manuals for many years.

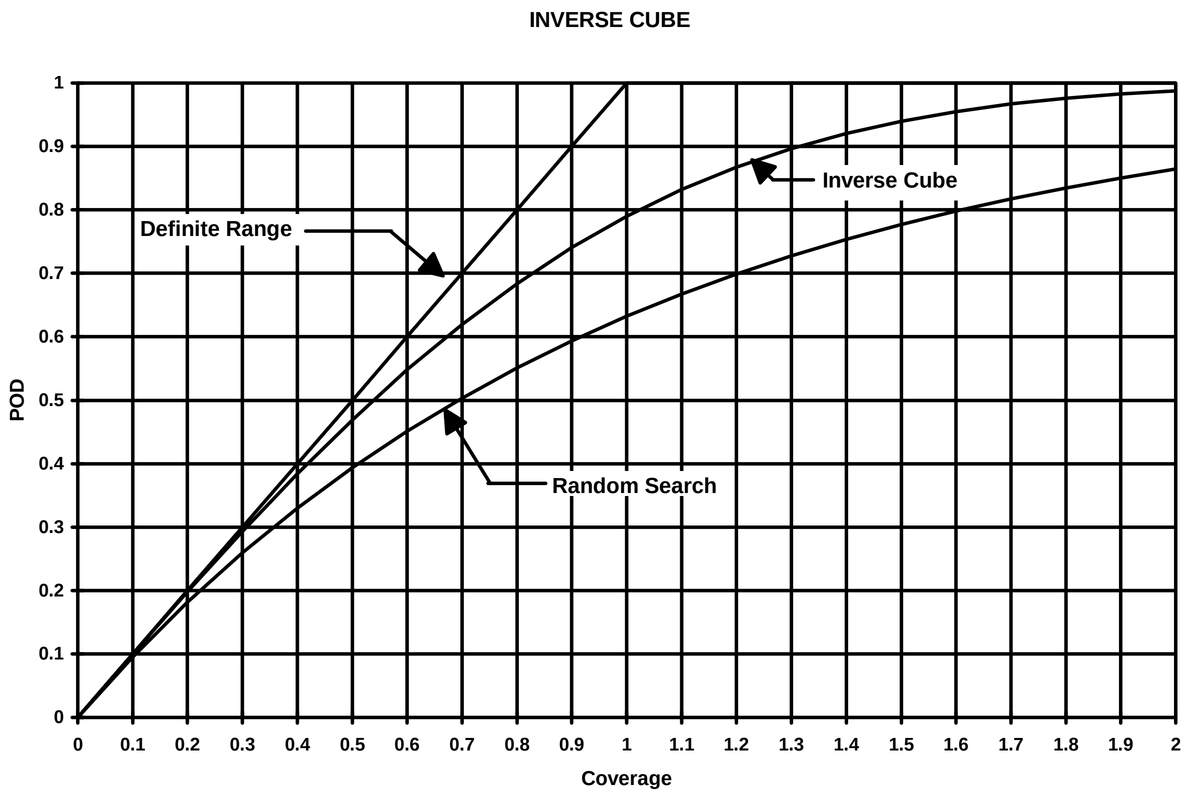

Figure 5-8

The simplified search planning method (SSPM) assumes an inverse cube sensor will be used in a perfect parallel sweep search relative to the search object. Furthermore, the POD versus Coverage curve used with the SSPM is based on this assumption.

5.8.3 With respect to a graph of the three curves depicted in Figure 5-8, Koopman observed,

> *At one extreme is the case of the definite range law, at the other the case of random search. All actual situations can be regarded as intermediate curves ... The inverse cube law is close to a middle case, a circumstance which indicates its frequent empirical use, even in cases where the special assumptions upon which its derivation was based are largely rejected.*

In these statements, Koopman's "*All actual situations* ... " clearly do not include those where systematic errors distort the search pattern relative to the search object even though such situations are common in actual operations. For example, search patterns designed for motionless search objects are almost always employed by search facilities when searching for objects, such as those adrift on the ocean, which are known to be moving. Under some circumstances, plotting these patterns relative to the moving object shows that the area covered is very different from that which should have been covered, with disastrous effects on the POS. However, that is another topic beyond the scope of this paper.

5.8.4 In summary, the inverse cube model of visual detection has the following advantages.

- The inverse cube model is based on a representation of the geometry of the operational situation.
- The POD function for parallel sweep searches using an inverse cube sensor can be expressed in terms of an existing and well-known mathematical function, namely the error function $erf(x)$.
- The POD curve falls approximately midway between the "extremes" of the POD curve generated by parallel sweep searches with definite range sensors, and that of random search.

The disadvantages of the inverse cube model include the following.

- Its validity as a model of visual detection has never been confirmed for any situation.
- Three of the four basic assumptions upon which the model is based are clearly not true for SAR situations.
- Even if the inverse cube model is assumed to be valid, the validity of its POD function depends upon a search pattern that has perfectly parallel,

equally spaced tracks relative to the search object, a situation that rarely if ever exists in practice.

5.9 Search Effort, Coverage and POD. The differences among the three curves shown in Figure 5-8 may not appear to be particularly large. However, consider the following. From equation [5-3], it is easy to see that for any area of size *A*, the search effort *Z* required to cover it with a coverage of *C* is proportional to *C*. That is, it takes twice as much effort to get twice the coverage of the same area. Now suppose a search planner wants to put enough search effort into an area to achieve a POD of 79%. Looking at the graphs in Figure 5-8, it is seen that this can be done by a definite range sensor at a coverage of 0.79. To get the same result with an inverse cube sensor requires a coverage of 1.0 which is nearly 27% greater. This means it will take 27% more effort to get the same 79% POD with an inverse cube sensor as it did with the definite range sensor. With random search, a coverage of 1.56 is required to achieve a 79% POD. This is almost twice the coverage (a 97% increase) required for a definite range sensor and is 56% more than the coverage required for an inverse cube sensor. The differences in the amounts of search effort required to achieve a given result are clearly much larger than the differences in the sensors' POD functions.

> The amount of search effort required to achieve a certain POD with a parallel sweep search is highly dependent upon, and sensitive to, the nature of the sensor's lateral range curve.

For this reason, it is very important to have accurate knowledge about both the shape of the sensor's lateral range curve and its associated sweep width under the actual search conditions encountered.

5.10 Effects Leading to Random Search PODs. In this chapter it has been shown that the random search POD curve is a good estimator in two quite distinct situations.

1) Whenever any sensor follows a random path within the search area, the random search POD curve should be used.
2) Whenever the lateral range function, $p(x)$, has a low maximum value at $x = 0$ and maintains a low but nearly constant value over the interval $(-x,+x)$ where x is large compared to the sweep width W, the random search POD curve should be used.

Of course, this immediately raises the following two questions.

a) Just how "random" does a searcher's path have to be before POD is significantly impacted?

b) Just how low and flat does a sensor's lateral range curve have to be before its parallel sweep POD is essentially that of random search?

5.10.1 One approach to answering the first question is to reason as follows. Suppose a large number of experiments were done to determine the lateral range function where the ranges and bearings of all detections (and misses) from the sensor's *intended position* on the *relative track* were accurately recorded but the exact positions of the sensor with respect to the search object were not known. The resulting lateral range curve would then represent the "average" or "expected" lateral range function with respect to the intended relative track and would reflect the combined effects of the sensor's true lateral range function and the probable error of position relative to the search object.

5.10.2 The probable error of position relative to the search object includes probable errors in both the search facility's position and the search planner's estimates of the search object's position. This is important because the quality of the search pattern depends on how accurate it is *relative to the search object*. It is reasonable to assume the degree of "randomness" in the relative track is represented by the size of the probable error of position relative to the search object. The distribution of position errors is generally assumed to be circular normal. This means the distribution of errors perpendicular to the *relative track* (*i.e. the distribution of cross-track errors)* is normal. If it is assumed that the search legs are long compared to the sweep width while the spacing between them is roughly the same as the sweep width, then the cross-track error is far more important than the component along the track. Thus it is reasonable to consider only the effects of cross-track error in estimating the "average" or "expected" lateral range curve.

5.10.3 If the true (no position error) lateral range function is known or such a function has been postulated, and the probable error of position relative to the search object can be estimated, it is then possible to estimate the expected lateral range function with respect to the intended relative track by computing the *convolution integral* of the true lateral range function and the normal (cross-track error) function as follows:

[5-14] $$\varphi(x) = \int_{\infty}^{+\infty} p(xt)\mathrm{Z}(t,\sigma)dt$$

where _(*x*) is the expected probability of detection at a lateral range of *x* from the intended relative track, *p*(*x-t*) is the probability of detection at the true lateral range *x-t* from the sensor, and Z(*t,σ*) is a normal distribution of cross-track errors *t* having a standard deviation of σ. The general form of a normal distribution Z with a mean (average) of zero and a standard deviation of σ is

[5-15] $$Z(t,\sigma) = \frac{e^{\frac{t^2}{2\sigma^2}}}{\sigma\sqrt{2\pi}}.$$

5.10.4 Several properties of the convolution integral in [5-14] should be noted. First, the sweep width of the expected lateral range function is identical to the sweep width of the true lateral range function. That is,

[5-16] $$\int_{\infty}^{+\infty} \varphi(x)dx = \int_{\infty}^{+\infty} p(x)dx = W.$$

Second, as the value of σ increases, the expected lateral range function becomes less "peaked" and more spread out as compared to the actual lateral range function. The effect is something like pressing down on the center of a semi-rigid closed container full of water whose original cross-sectional shape is like that of the true lateral range curve. As the pressure (σ) increases, the center becomes lower while the extremities ("tails") become higher to accommodate the displaced volume of water.

As a practical matter, by the time the value of σ is as large or larger than the sweep width *W*, the random search POD curve should be in use.

5.10.6 So far, only the first question about the impact of "randomness" in the relative search track has been directly addressed. The second question about just how "flat" a lateral range curve must get before the random search curve is effectively reached has been indirectly addressed because accounting for the uncertainty about the relative search track has the effect of "flattening" the expected lateral range curve. Unfortunately, there is no measure of "flatness" readily available. The only alternative is to evaluate each lateral range function in a parallel sweep search scenario and compare the resulting POD curve with that of random search.

5.11 <u>Search Conditions</u>. Recent experiments done by the U. S. Coast Guard Research and Development Center indicate that deteriorating search conditions, such as decreasing visibility, increasing sea states, increasing searcher fatigue, etc. appear to have two effects. First, the sweep width becomes smaller as conditions deteriorate; an effect that has been known for many years. Second, the lateral range function tends to become more "flat" (less "peaked"), an effect that, though apparent, still has not been fully analyzed and incorporated into operational search planning methods. Figure 5-9 depicts the effects on an inverse cube lateral range curve of both sweep width reduction and “flattening” due to deteriorating conditions and navigational error. The only known attempt to date to modify operational search planning to accommodate this second effect is a recent development that provides both the inverse cube and random search POD curves with somewhat vague instructions to use the former when search conditions are “ideal” and the latter when search conditions are “poor.” (Actually, a little more guidance is provided by suggesting a comparison between “uncorrected” sweep widths (ideal conditions) and “corrected” sweep widths (less than ideal conditions) based on the size of the difference between the two.) However, as more research and analysis is done, it may be appropriate to provide more than just these two POD curves to choose from. Recalling from paragraph 5.9 above that the required level of search effort to attain a given POD is very sensitive to small changes in the POD curve, the use of intermediate POD curves deserves consideration. However, another point of view is put forth in the next paragraph.

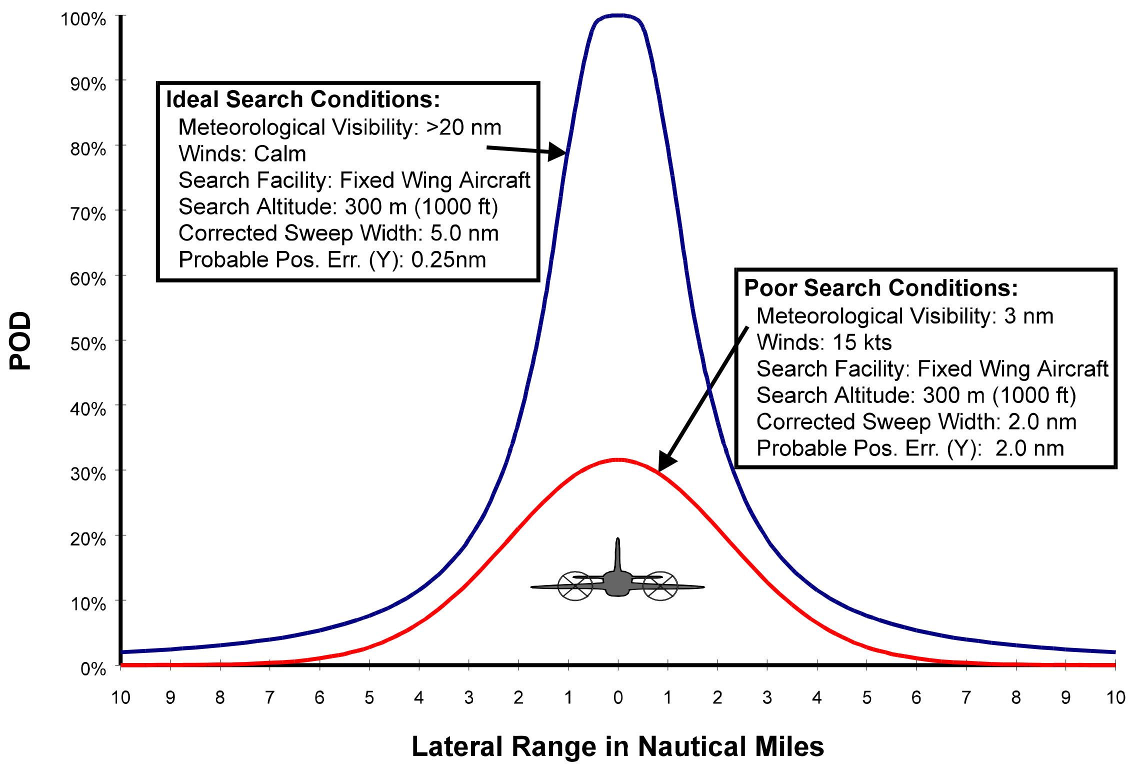

Figure 5-9

5.12 Navigation. While methods of navigation have improved dramatically in recent years with the advent of such systems as the Global Positioning System (GPS), the ability of search facilities to navigate with extreme precision is by no means universal. Besides, the probable position error of the search facility as compared to its intended location is only one factor in computing the probable position error with respect to the search object, the other factor being the uncertainty in the search object's position and movements. The probable relative position errors often encountered tend to be large in comparison to the sweep widths for typical SAR search objects, especially when the search facility is an aircraft. In addition, there are many other factors affecting how searchers and sensors perform under actual operational conditions as compared to "ideal" conditions. Uncertainty about the values of these factors have a similar impact on the "expected" lateral range curve, mathematically at least, to that just seen for uncertainty in the relative search track. This is because the use of convolution integrals is not limited to positional uncertainties. A case may still be made for the random search POD curve being the most realistic representation of the POD values which should be expected under operational conditions.

Chapter 6
Optimal Search Plans

6.1 Introduction. Chapter 2 introduced the concept that the goal of search planning is to maximize the *probability of success* (POS) which in turn depends on *probability of containment* (POC) and *probability of detection* (POD). Chapter 3 then explored several different search object location probability density distributions and described how to compute POC values for subareas within them. Chapter 4 introduced the concepts of *lateral range curves* and *sweep width* and used these concepts to describe several detection models. Chapter 5 went on to introduce the concepts of *search effort* and *coverage* and then examined how the various models described in Chapter 4 perform when used in parallel sweep searches by developing their parallel sweep POD curves as functions of coverage. Now that methods for computing actual POC, search effort, coverage, and POD values have been developed, it is time to bring all of these concepts together to formulate a theory of *optimal search*. As first mentioned in Chapter 2, an optimal search is one in which the highest possible POS is achieved with the available search effort. This chapter will first examine how to optimally allocate search effort over a uniform distribution of possible search object locations since that is the simplest possible optimization problem. Next, the problem of optimizing search effort allocation over a simple two-part non-uniform distribution will be studied. The optimal allocation of search effort over a circular normal distribution will be examined in some detail since that is the distribution of possible search object locations on which the simplified search planning method (SSPM) is based. Finally, the additive principle of optimal search is presented followed by a brief discussion of other factors besides POC and POD which may warrant inclusion in the computation of POS.

6.2 Uniform Search Object Location Probability Density Distributions. In a uniform distribution of possible search object locations within some finite area A, the POC for the entire area A is 1.0 (100%). The POC value for any subarea a_i in A is simply a_i/A. Suppose the amount of available search effort Z is equal to $A/2$. With this amount of effort, it is possible to search the entire area A with a coverage of 0.5. It is also possible to search one-half of A with a coverage of 1.0, one-third of A with a coverage of 1.5, etc. There are infinitely many possibilities, assuming both the search effort and search object location probability distribution are infinitely divisible (a luxury not available to search planners in the real world but nevertheless quite useful to theorists). The question optimal search theory tries to answer is, “Which combination of area and coverage will produce the highest POS?”. The simplest way to start looking for an answer to this question is to perform a few trials. However, before answering this question, a few more assumptions will be made and maintained for the remainder of the chapter. It will be assumed that all searches employ equally spaced parallel sweeps relative to the search object. It will be further assumed that only one contiguous area is searched and that the search effort is uniformly distributed throughout that area, i.e. the same coverage will be used in all parts of the area. This last assumption is not entirely consistent with the first assumption of parallel sweeps due to the shapes and the overlapping of lateral range curves described in the last chapter. However, at the coverages being evaluated in the trials to follow, this assumption is a reasonable approximation of the true situation.

6.2.1 Definite Range. If a definite range sensor is used, then searching the entire area *A* with a coverage of 0.5 will produce the following results. The POC will be 1.0 since all of *A* is included. From Figure 5-8 it is seen that the POD for a definite range sensor at a coverage of 0.5 is also 0.5. This makes the POS for this trial

$$POS = POCxPOD = 1.0x0.5 = 0.5$$

or 50%. For the second trial, assume one-half of *A* is searched with a coverage of 1.0. This makes the POC 0.5 and the POD 1.0. Computing the POS,

$$POS = POCxPOD = 0.5x1.0 = 0.5$$

which is again 50%. For the third trial, assume one-third of *A* is searched with a coverage of 1.5. This makes the POC 0.33 but the POD remains at 1.0 as it does for this sensor for all coverages greater than or equal to 1.0. This time, the POS is

$$POS = POCxPOD = 0.33x1.0 = 0.33$$

or about 33%. For this sensor and probability distribution, the same amount of effort will produce the same POS as long as the coverage is less than or equal to 1.0. So, for search areas within the range of *A* down to *A*/2, any decrease in search area is exactly matched by the increase in POD that comes with the increase in coverage. For areas less than *A*/2, coverages become more than 1.0 but since the POC continues to decrease while there is no further increase in POD, such high coverages actually reduce the search's chances of success.

6.2.2 Inverse Cube. If the sensor is changed from a definite range type to an inverse cube type while keeping all other things equal, then the POC values and coverages will remain the same as in subparagraph 6.2.1 above. However, the change of sensor will be reflected in a change in the POD values at the various coverages. Repeating the first trial, the POC remains 1.0 but the POD from the inverse cube curve in Figure 5-8 is now only 0.47. Computing the POS with these values,

$$POS = POCxPOD = 1.0x0.47 = 0.47$$

or about 47%. For the second trial, the POC is 0.5 and the coverage is 1.0. From Figure 5-8, the inverse cube POD for this coverage is 0.79. The POS for this trial is

$$POS = POCxPOD = 0.5x0.79 = 0.395$$

or about 39.5%. Note that this is substantially less than the POS of the previous trial. For the third trial, the POC is 0.33 while the coverage is 1.5. Again consulting Figure 5-8, the POD for an inverse cube sensor at this coverage is found to be about 0.94. Computing the POS,

$$POS = POCxPOD = 0.33x0.94 = 0.313$$

or about 31.3% which is even smaller than the previous trial. It is clear from these trials that when an inverse cube sensor is used to search a uniform distribution of possible search object locations, the amount of area searched with the available search effort should be maximized at the expense of coverage in order to achieve the highest possible probability of success. It is easy to see why this is true by comparing the inverse cube POD curve to the definite range POD curve. For coverages less than or equal to 0.5, doubling the coverage doubles the definite range POD. Since the slope of the inverse cube POD curve gradually decreases from 1.0 at the origin to 0 as the coverage increases without bound, increases in the coverage produce smaller and smaller increases in POD as the coverage becomes greater and greater. So, for uniform distributions, decreasing the search area decreases POC faster than the increase in coverage can increase an inverse cube sensor's parallel sweep POD. The net result is a loss in POS.

6.2.3 Random Search. Repeating the above trials using the random search curve to obtain POD values produces results similar to the inverse cube results, only lower. The POD for the first trial is about 39%, for the second about 63% and for the third about 78%. The corresponding POS values are about 39% for the first trial, 31.5% for the second trial and about 26% for the third trial. Again, the conclusion is that for a uniform search object location probability density distribution, the amount of area searched should be maximized at the expense of the coverage.

6.3 A Simple Non-Uniform Distribution. Consider a square area measuring 5 units on a side which has a POC for a given search object of 1.0 (100%). If the possible search object locations were uniformly distributed, the probability density (ρ) everywhere in *A* would be the same as the average value

$$\rho = \frac{POC}{A} = \frac{1.0}{25} = 0.04$$

or 4% per square unit of area. Consider a second square, concentric to the first, which measures 3 units on a side. The original 5 x 5 square is now divided into two regions — the region contained inside the smaller square, and the region which is outside the smaller square but inside the larger one. Figure 6-1 depicts this situation. Now suppose the POC for the inner (smaller) square is 0.5 (50%) and is uniformly distributed within that square. This leaves the remaining 50% of the search object's possible locations in the region between the inner and outer squares. Suppose that within this region, the possible search object locations are uniformly distributed. The area (a_1) of the small square is 3 x 3 = 9 square units which makes the search object location probability density ρ_1 = 0.5/9 = 0.0556 or about 5.56% per square unit. The area of the other region is 25 - 9 = 16 square units making the probability density ρ_2 = 0.5/16 = 0.03125 or about 3.125% per square unit. Given these values, it is instructive to begin with trial searches similar to those done in the previous paragraph and then try some other variations to see how it might be possible to determine the optimal area to search. Again it will be assumed that the available search effort is equal to one half of the possibility area. In this case, that would be 25/2 or 12.5 square units. It will be assumed that all trial search areas are squares which are concentric with the possibility area.

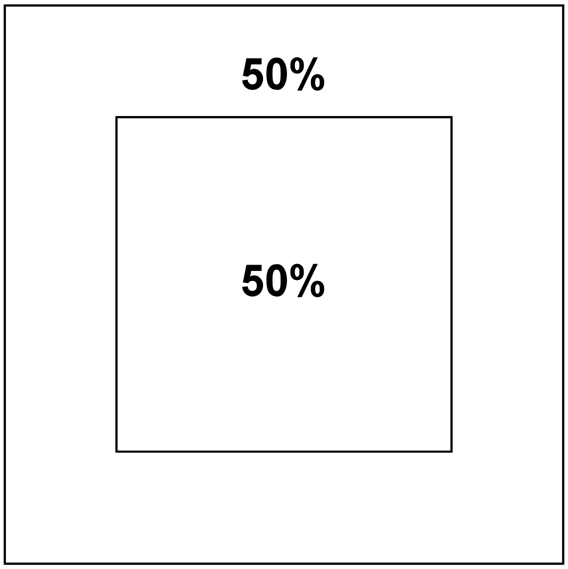

Figure 6-1

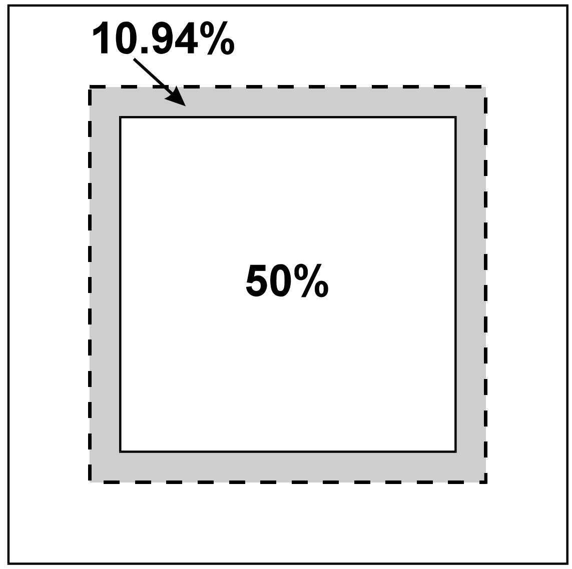

Figure 6-2

6.3.1 Definite Range. Just as in the first trial of subparagraph 6.2.1, searching all of *A* at a coverage of 0.5 will produce a POS of 0.5 (50%). In the next trial, one-half of *A* was searched at a coverage of 1.0. The length of one side of a square with one-half the area of the possibility area is computed as

$$\sqrt{\frac{25}{2}} = \frac{\sqrt{25}}{\sqrt{2}} = \frac{5}{\sqrt{2}} = \frac{5\sqrt{2}}{2} \approx 3.54$$

which is slightly larger than the 3 x 3 central square. Thus the POC for the search area is the POC for the central square ($POC_1 = 0.5$) plus some additional probability from the other region. To find out just how much additional probability, the amount of area outside the central square but inside the search area (the shaded region in Figure 6-2) needs to be computed. Since the area of the search area is 12.5 square units and the area of the inner square is 9 square units, the area we are seeking is 12.5 - 9 = 3.5 square units. The probability density in this region was previously computed to be 0.03125 so the amount of additional probability is

$$POC_2 = \rho_2 x\, a_2 = 0.03125x3.5 = 0.1094$$

This makes the total POC for the search area 0.5 + 0.1094 = 0.6094 or about 61%. Computing the POS for this search,

$$POS = POCxPOD = 0.6094x1.0 = 0.6094$$

or about 61%. In this case, decreasing the area searched and increasing the coverage significantly improved the POS over the previous trial, a result quite different from when the same two trials were performed with a completely uniform distribution of possible search object locations. To perform the third trial where only one-third of the possibility area is covered with a coverage of 1.5, it is necessary to compute the dimensions of the third search area as follows:

$$\sqrt{\frac{25}{3}} = \frac{\sqrt{25}}{\sqrt{3}} = \frac{5}{\sqrt{3}} = \frac{5\sqrt{3}}{3} \approx 2.89\,.$$

This value is smaller than the dimensions of the inner square (3 x 3) so this third trial search area will be contained entirely within it. The area of the third trial search area is 25/3 or about 8.33 square units. The POC for this area is

$$POC = \rho_1 x\, a_3 = 0.05556x8.33 = 0.4630$$

Computing the POS,

$$POS = POCxPOD = 0.4630x1.0 = 0.4630,$$

or about 46% which is less than either of the previous trials. The fact that over the course of these trials, which progressed from larger areas and lower coverages to smaller areas and higher coverages, the POS went from a lower value to a higher value and then back down to a lower value again suggests that there exists somewhere between the two search areas and coverages that gave the lower POS values, there is an optimum search area and coverage which produces the highest possible POS for the given amount of search effort. Clearly the optimal area cannot be smaller than 12.5 square units since increasing the coverage for a definite range sensor above 1.0 reduces POC without increasing POD and hence reduces POS. As it turns out, increasing the search area and reducing the coverage to a value below 1.0 also decreases the POS as any trial using this two-part distribution will show. Hence, the optimal coverage to use with a definite range sensor in this case is 1.0, assuming the search effort is properly placed so that the areas with the highest probability density are covered.

6.3.2 Inverse Cube. Performing the same trials as in 6.3.1 above with an inverse cube sensor produces the following results.

- Trial 1: POS = POC × POD = 1.00 × 0.47 = 0.47 or 47%
- Trial 2: POS = POC × POD = 0.61 × 0.79 = 0.48 or 48%
- Trial 3: POS = POC × POD = 0.46 × 0.94 = 0.43 or 43%

In this case also, searching with a coverage of 1.0 produces the optimal result.

6.3.3 Random Search. Performing the same trials as in 6.3.1 above using random search produces the following results.

- Trial 1: POS = POC × POD = 1.00 × 0.39 = 0.39 or 39%
- Trial 2: POS = POC × POD = 0.61 × 0.63 = 0.38 or 38%
- Trial 3: POS = POC × POD = 0.46 × 0.78 = 0.36 or 36%

For random search, maximizing the area at the expense of coverage is still the best policy, but not dramatically so.

6.4 Optimal Search Factors. The trials performed in paragraphs 6.2 and 6.3 above show that the optimal search area and coverage depends on both the nature of the distribution of possible search object locations and on the nature of the sensor employed. If both are known or assumed, it is then possible to develop *optimal search factors* based on the size of the available search effort relative to the “size” of the search object’s location probability density distribution. For point datums, the total probable error of position will be used to measure the “size” of the search object location probability distribution. The optimal search factor will then be the value which, when multiplied by the total probable error of position, produces the optimal search radius. Circumscribing a square about a circle of this radius centered on the datum point will produce the optimal search area. Optimal search factors will be explored in the next paragraph where the search object location probability distribution is assumed to be of the circular normal type commonly used with point datums and the sensor is assumed to be represented by the inverse cube parallel sweep POD curve. All of these are assumptions on which the SSPM is based.

6.5 The Circular Normal Distribution. The circular normal distribution was described in Chapter 3. With this distribution, it is not possible to define the boundary of the possibility area. This is because the probability density never (in theory) actually becomes zero, even though it becomes vanishingly small as the distance from the distribution’s center becomes very large. However, it is possible to define the boundary of the smallest area which contains 50% of the distribution. As shown in Chapter 3, this boundary is a circle with a radius of about 1.1774 standard deviations (1.1774σ). This is also, by definition, the value of E, the total probable error of position. If E is taken to be the basic unit of measure, then a square measuring $2E$ on a side and centered on the distribution will have a POC of 0.5791 or about 58%. This is the circumscribed square described in subparagraph 3.3.4 of Chapter 3. The area of this square is $4E^2$ and so, for convenience, it will be assumed in the trials that follow that the available effort is also $4E^2$.

6.5.1 Inverse Cube. The coverages used in the previous trials were 0.5, 1.0 and 1.5. To find the amount of area which can be searched at a given coverage with a given effort, it is necessary to solve equation [5-3] for A as follows.

[6-1] $$A = \frac{Z}{C}$$

Using this formula, it is seen that with $4E^2$ units of effort, an area of $8E^2$ square units can be searched at a coverage of 0.5. A square of this area measures about $2.828E$ units on a side. The radius of a circle inscribed within this square is half this value or $1.414E$. Recalling that $E = 1.1774\sigma$, this radius is equivalent to 1.665 standard deviations. From the standard normal tables in Appendix A, the joint probability that both coordinates of the search object’s location relative to the center lie between plus and minus 1.665 standard deviations is

$$\left[2(0.95200.5)\right]^2 = 0.8172$$

or about 82%. Using this POC value and the inverse cube's parallel sweep POD of about 0.47 (47%) for a coverage of 0.5, the POS is computed as

$$POS = POCxPOD = 0.82x0.47 = 0.3841$$

or about 38%. For the next trial, the search area is equal to the effort ($4E^2$) so the square area searched will measure 2*E* on a side and the radius of the inscribed circle will be *E*. The POC for this square has already been computed as 0.5791 or about 58%. The inverse cube parallel sweep POD for a coverage of 1.0 is about 79%. Using these values to compute POS,

$$POS = POCxPOD = 0.58x0.79 = 0.4575$$

or about 46%. Finally, the area which can be covered at a coverage of 1.5 is computed from [6-1] as $2.6667E^2$. A square with this area measures 1.633*E* on a side with an inscribed circle having a radius of 0.8165*E*. This is equivalent to 0.9613 standard deviations. The POC for this square, as computed using the standard normal tables, is

$$\left[2(0.83180.5)\right]^2 = 0.4405$$

or about 44%. The parallel sweep POD for an inverse cube sensor at a coverage of 1.5 is about 94%. Computing the POS,

$$POS = POCxPOD = 0.44x0.94 = 0.4140$$

or about 41%. Again, out of the three trials, a coverage of 1.0 gives the highest POS. However, it is not certain whether this is the highest possible POS which can be obtained with $4E^2$ units of effort.

6.5.2 Optimal Inverse Cube Search Area. Unfortunately, there is no way to directly compute the size of the optimal search square. That is why the method of successive trials has been used up to now even though the computations were rather cumbersome and tedious. Fortunately, for problems where repetitive trials are preformed using incremental changes in the input parameters to search for a maximum result, we have computers. A computer program was written which expands the search square by increments until the computed POS begins to decline. The program then goes back to the square computed two steps earlier, cuts the increment in half, and repeats the process. This procedure is repeated until the increments become very small. Using this program to find the optimum search square for $4E^2$ units of search effort produces the following results. A square measuring 2.04*E* on a side (optimal search factor = 1.02) should be searched, making the POC 0.5940, the coverage 0.96, the POD 0.7707, and the POS 0.4578. Again, the optimal coverage is very close to 1.0. It is little wonder that the developers of the SSPM seemed to consider this level of coverage ideal.

6.5.3 Optimizing Other Levels of Available Effort. In all the trials so far, the amount of available effort was always enough to cover 50-60% of the distribution at a coverage of 1.0 and in most cases using that coverage produced an optimal or nearly optimal result. However, the availability of search effort depends on the availability of search facilities while the size of the (practical) possibility area depends on the amount of uncertainty (total probable error of position) about the search object's location. These two quantities, available effort and possibility area, are largely independent so there is no guarantee that a search planner will have, or can get, just the amount of effort required to cover 50-60% of the possible locations with a coverage of 1.0. Consider a circular normal distribution where the available effort is only $2E^2$. At this level of effort, the computer program produces an optimal search factor of 0.81 and thus recommends a square measuring 1.62*E* on a side be searched. This makes the POC 0.4356, the coverage 0.76, the POD 0.66, and the POS 0.2875. Using the same amount of search effort to search a smaller square 1.1414*E* on a side at a coverage of 1.0 produces a POS of only 0.2140. Next, consider the situation if $8E^2$ units of effort are available. This time, the computer program computes an optimal search factor of 1.29, recommending a square 2.58*E* units on a side be searched. This makes the POC 0.7571, the coverage 1.21, the POD 0.8701, and the POS 0.6588. Searching a larger square 2.828*E* units on a side at a coverage of 1.0 produces a POS of only 0.5610. It is clear from these last two examples that using the available effort at a coverage of 1.0 does not always produce optimal results.

6.6 The Additive Principle of Optimal Search. Up to now, all discussions of optimal search have considered only single search efforts. Operationally, it is often necessary to perform several searches before the search object is located. A question of both theoretical and practical interest is whether there would be any advantage to applying the total search effort consumed over several searches to a single, massive search. One of the more important theorems of search theory provides the answer.

> Given an amount of search effort Z, the maximum cumulative probability of success may be attained by either a single optimal search using all of Z or by smaller, individually optimized searches using efforts z_i, where $Z=\Sigma z_i$, provided each of the searches is optimized with respect to an updated version of the

search object location probability density distribution which accounts for the effects of all previous searching.

6.6.1 This theorem opens the door for developing a single set of optimal search factors, based on cumulative search effort, which may be used to compute the optimal search square for either a single search or each of several searches in a series. The computer program cited in paragraph 6.5.2 was used to compute and graph optimal search factors for both the inverse cube and random search functions over a circular normal distribution of possible search object locations. Figures 6-3 and 6-4 show these *optimal search factor curves*, where *relative effort* is defined as the actual available search effort in square nautical miles divided by the square of the total probable error (the *effort factor*) and the *cumulative relative effort* is the sum of all the individual relative effort values over all searches, including the one which is being planned.

6.6.2 It is also possible to develop cumulative POS curves based on the cumulative relative effort expended to date and the assumption that all searching over the circular normal distribution has been optimal. Figure 6-5 shows the cumulative POS curves for optimal searches around point datums for both the inverse cube model and random search.

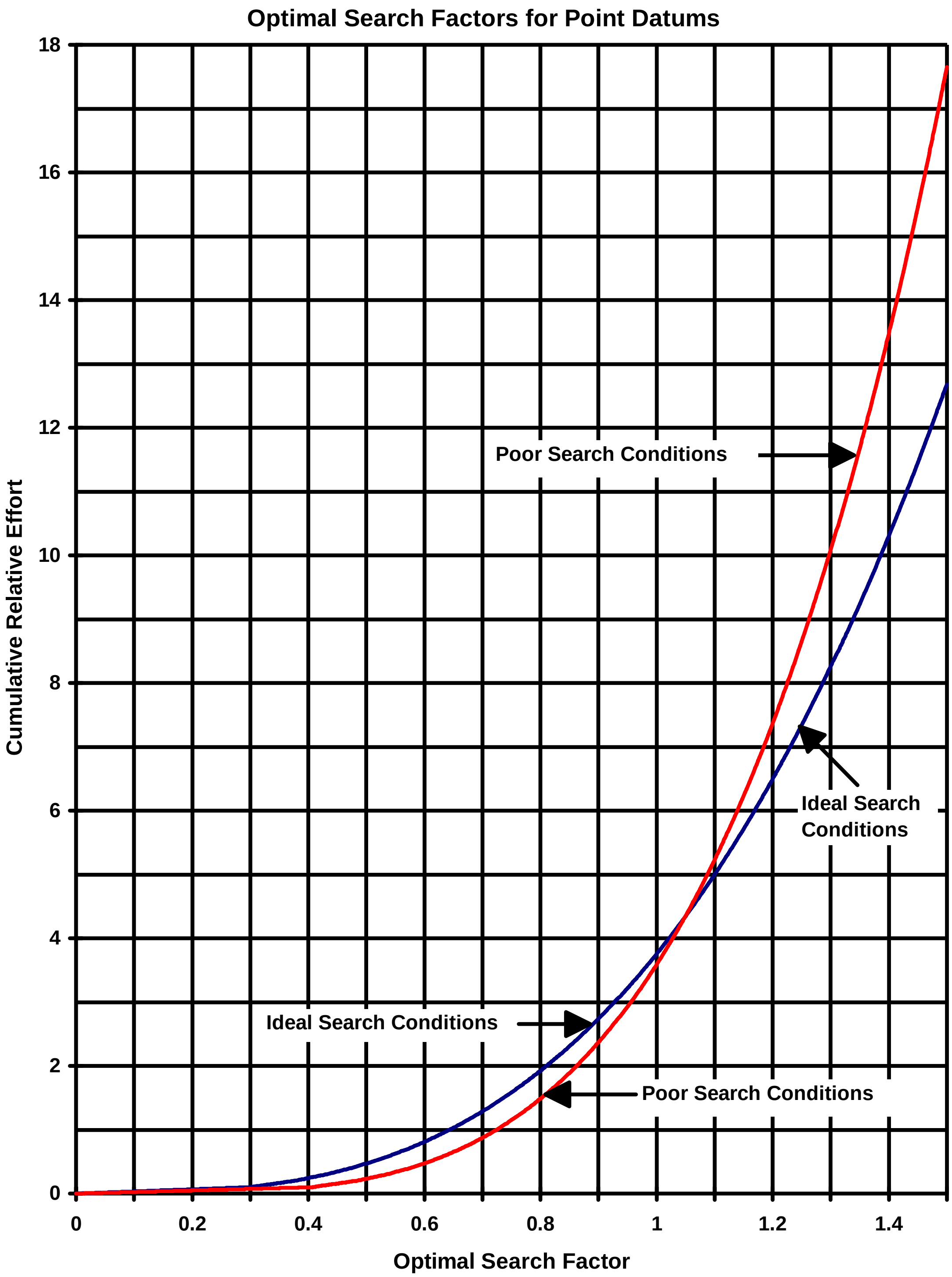
Optimal Search Factors for Point Datums
Cumulative Relative Effort
18
16
14
12
10
8
6
4
2
0
0
0.2
0.4
0.6
0.8
1
1.2
1.4
Optimal Search Factor
Poor Search Conditions
Ideal Search Conditions
Ideal Search Conditions
Poor Search Conditions

Figure 6-3

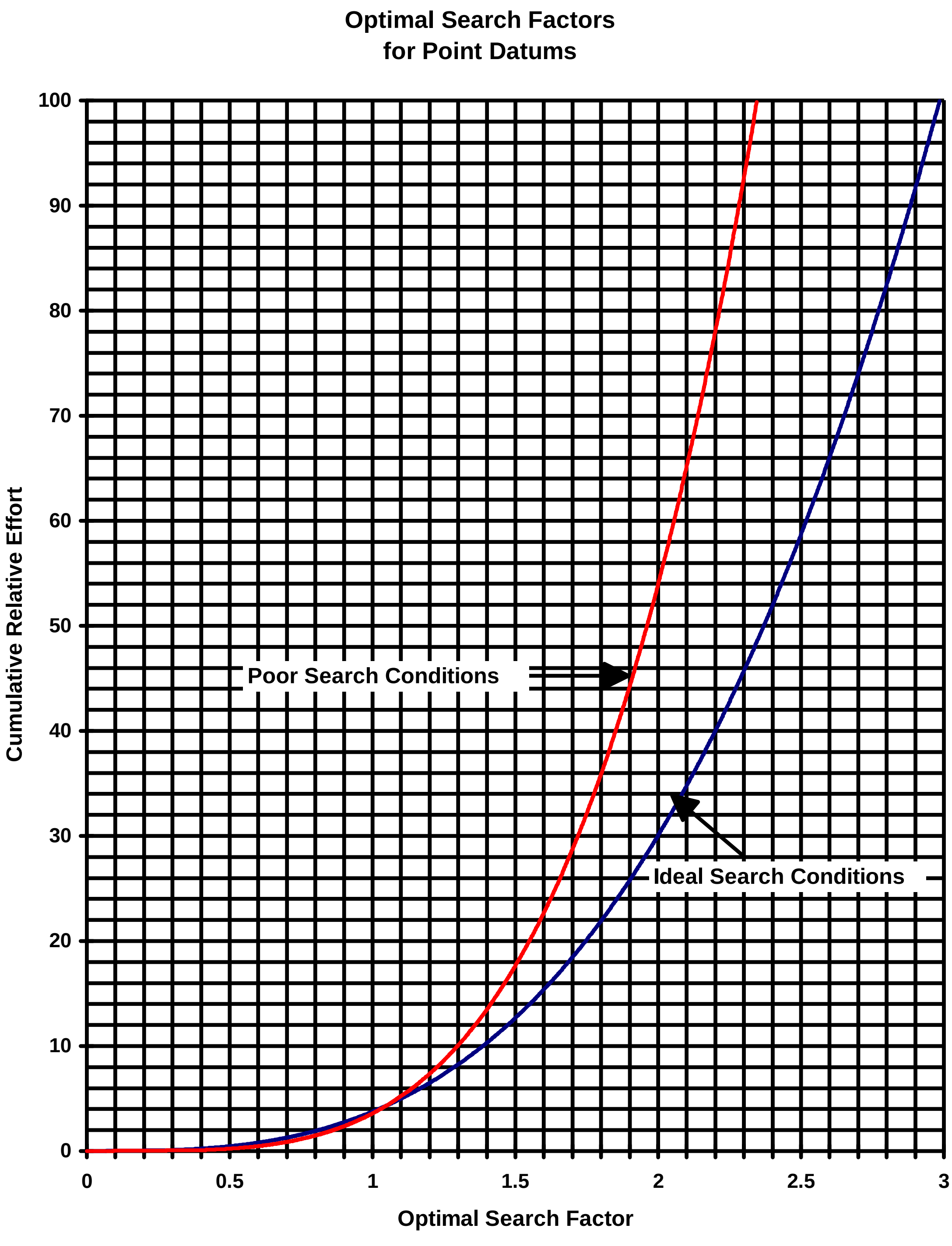
Optimal Search Factors
for Point Datums
100
90
80
70
60
50
40
30
20
10
0
Cumulative Relative Effort
Poor Search Conditions
Ideal Search Conditions
0
0.5
1
1.5
2
2.5
3
Optimal Search Factor

Figure 6-4

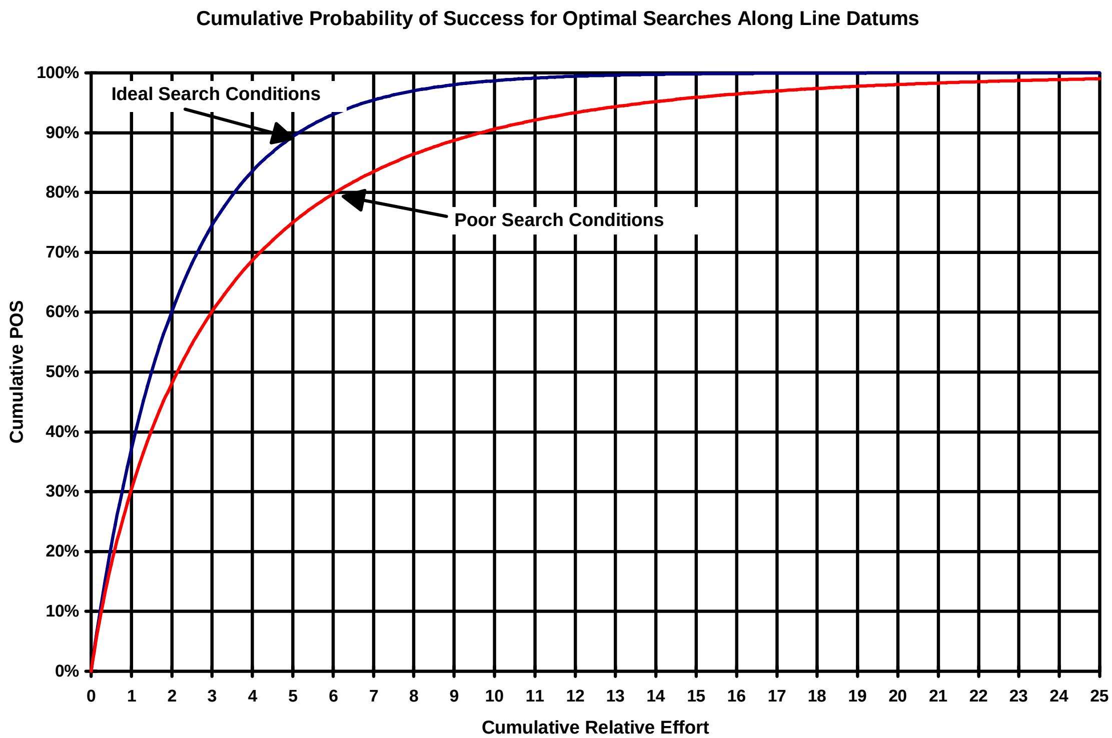

Figure 6-5

6.6.3 There is a small inconsistency between the theory and practice of optimal search planning. The theorem stated in paragraph 6.6 requires optimal allocation of effort on all searches. To be truly optimal, the coverage of a non-uniform distribution of possible search object locations should vary with the probability density. That is, high coverages should be used in subareas where the probability density is high and low coverages should be used in areas where the probability density is low and relatively uniform. This is impractical in real-world operations. The probability density of a circular normal distribution varies continuously as distance from the center increases. It is not operationally possible to vary the coverage continuously to match the probability density. However, ***if*** it could be done, then as the cumulative relative effort increased, whether for one or many searches in a series, the optimal search radius would also increase, *regardless of whether the available effort was greater or less than the effort used in the previous search.*

6.6.4 To see why the optimal search radius always increases with truly optimal searching, consider the following. Starting with a small area centered on the "peak" of a circular normal distribution, the optimal coverage would be that which reduces the density in this small central area to the density of the small areas immediately surrounding it, forming a small plateau. The next optimal search would reduce the density in this small plateau to that of the immediately surrounding area, thus enlarging, and lowering, the plateau of equal density. This technique, if repeated until the available search effort is expended, would produce the highest possible POS by reducing the plateau of equal density to the lowest possible level. Under these circumstances, there would never be a situation where a subarea of an area that had been searched had a higher density than some other part of the searched area. That is, the density within the area searched to date would always be uniform. On the other hand, using a uniform coverage over a large part of a circular normal distribution will leave a “cliff” around the edges of the searched area where the density inside has been reduced to a level lower than that immediately outside the searched area. Also, the density within the searched area will not be uniform. It will be a reduced version of the original distribution, including a central “peak” of the highest density remaining in the searched area.

6.6.5 Using the optimal search factor curves in Figures 6-3 and 6-4 which are based on uniform coverages of large areas also produces monotonically increasing optimal search radii. However, if, in a series of large-area optimal searches using uniform coverage, the available relative effort for the next search is much, much less than its predecessors, searching the recommended still larger area will not be optimal. Instead, the optimal area to search with such a small effort will be the remaining central “peak” where the highest probability density lies. Fortunately, this situation is rare in actual search operations. Figure 6-6 is a scatter plot of optimal search factors for each search in a series of 5 searches where the effort relative effort of any one search was picked at random from a uniform distribution between 0.1 and 50.0. Two hundred fifty groups of 5 sequential searches each were examined and the vast majority of optimal search factors clustered around the optimal search factor curve. However, a few did fall well off the curve and they represent those situations where the reduction in the available effort for the next search was severe enough to cause a significant departure from the computed optimal search factor curve.

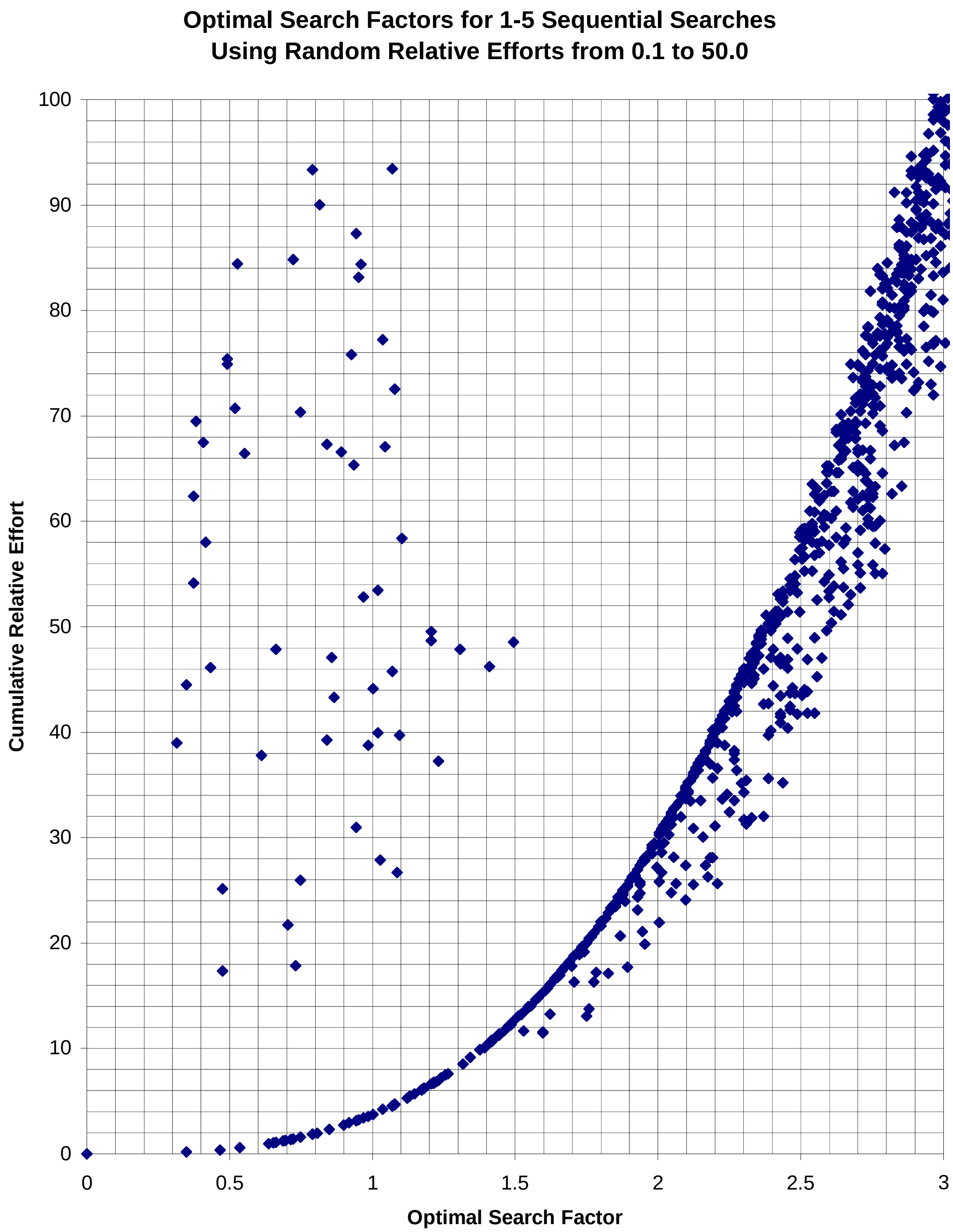
Optimal Search Factors for 1-5 Sequential Searches
Using Random Relative Efforts from 0.1 to 50.0
Cumulative Relative Effort
100
90
80
70
60
50
40
30
20
10
0
0
0.5
1
1.5
2
2.5
3
Optimal Search Factor

Figure 6-6

6.7 Other Optimization Criteria. This paper is based on the premise that the goal of search planning is to maximize POS and the assumption that POS depends solely on POC and POD. Although a conscious attempt was made to explicitly state all of the pertinent assumptions, some tacit assumptions remain. Chief among these are the following.

- The search object's location probability density distribution will remain a cohesive whole indefinitely and will neither expand too fast nor be torn asunder into multiple pieces.

- The survivors will stay alive until found, regardless of how long it takes.

6.7.1 If the first of these assumptions is not true, then neither is the additive principle of optimal search, at least not in practice. Consider the following example. There is a good deal of pleasure boat and aircraft traffic across the Straits of Florida between the U. S. mainland and the Bahama Islands. SAR cases in this area are common. However, if a boat or raft goes adrift between, say, Miami and Bimini, it will remain in the strait for only a couple of days before the Florida Current becomes the Gulf Stream at the strait's northern end. It is imperative that survivors be located and rescued before reaching the open Atlantic Ocean. At the north end of the strait, the strong Florida Current/Gulf Stream fans out, spins off eddies which can take on a life of their own for days or weeks, starts to meander, and generally makes predicting the movement of drifting objects very difficult. In any realistic simulation, the search object's probability distribution would be ripped apart with some of it moving rapidly northeast with the core of the Gulf Stream, some of it becoming involved with southerly counter-currents, possibly some of it becoming tied up in a warm-core or cold-core eddy drifting slowly away from the main current, etc. An optimization algorithm which "knew" ahead of time about how much search effort would be available each day and could "look ahead" and see the dispersion problem starting on the second or third day would concentrate the search effort in such as way as to minimize the probability of the search object escaping from the strait undetected, even if it meant sacrificing POS on individual searches early in the search effort in order to maximize the cumulative POS. Such an algorithm is impossible to implement without the use of computers. However, the Computer Assisted Search Planning system, version 2 (CASP 2.0) does have an algorithm capable of optimally allocating search effort over multiple searches based on predicted changes in the distribution of the search object's possible locations.

6.7.2 Often the ability of survivors to stay alive decreases rapidly with time. In this case, the appropriate optimization criteria might be minimizing the time to detect rather than maximizing POS over some longer period of time. Again, this requires a complex algorithm and a computer to implement. This capability is a planned enhancement for CASP 2.0 if it has not already been implemented.

Chapter 7
Conclusion

7.1 Introduction. The *improved search planning method* contains a “new” search planning methodology based on the concepts, theories, formulas and graphs developed in the preceding chapters, including the *optimal search factor* just described in Chapter 6. This methodology was developed to overcome several limitations of the previously published method. It is simply a more complete adaptation of basic search theory, as developed in the previous chapters of this paper, for practical use. In the paragraphs that follow, the merits of the older method with its fixed “safety factors” will be compared with the use of optimal search factors.

7.2 Limitations. The simplified search planning method (SSPM) has several serious limitations.

- It works correctly only for:

  → a point datum, where the search object location probability density distribution is assumed to be circular normal; and

  → a specific mathematical model of visual detection, called an “inverse cube law sensor,” of known sweep width following equally spaced parallel tracks under ideal search conditions.

- It used a fixed set of “safety factors” to determine the size of the “recommended” search area without regard to how much search effort was available to cover that area.

- It provided no guidance on how much search effort, or what coverage factor, should be used in the “recommended” search area.

- It provided no procedure for determining how to balance the conflicting goals of:
  → maximizing the size of the area searched; and
  → maximizing coverage of the area searched.

- It provided no measure of search effectiveness.

These limitations stem from the historical context in which search theory was first developed and then applied to practical situations.

7.3 Historical Note. As previously stated, modern search theory was developed in the 1940s during wartime, primarily for naval use. It was first applied to locating enemy naval forces in vast expanses of ocean, as well as to preventing the enemy from penetrating too closely to friendly forces undetected.

7.3.1 Not surprisingly, the first application of this new theory to search and rescue involved aviators forced down over the ocean. By the war's end, naval forces had become quite adept at locating and rescuing downed aviators; however, this specific SAR problem was relatively simple for the following reasons:

- The actual or planned movements of the aircraft involved were well known to those who would be responsible for any SAR efforts.

- Radio contact was usually maintained up until the pilot either bailed out or ditched. As a result, the time and place where the downed pilot went into the ocean were usually known within reasonably close limits, making a point datum an appropriate choice.

- The time between becoming distressed and the arrival of search units at the scene was relatively short, often only a matter of hours.

- Drift forces acting on floating objects in the open ocean were often approximately constant, and moderate, over the times and areas involved.

- There were usually substantial amounts of search effort available that were willingly employed to locate downed aviators.

7.3.2 This was fortunate, for the need to act quickly in wartime in isolated areas far from shoreside support and the lack of available computing power at that time meant the solution to the search planning problem had to be simple. Planning a search could involve only the barest minimum of manual computations. The complex body of search theory had to be reduced to a small and greatly oversimplified set of approximations for practical use in the field, resulting in the development of the SSPM. For its time and the problem it was intended to solve, the SSPM was superb.

7.4 Current SAR Environment. In contrast, today's civilian environment requires search planners to deal with many situations that are very different from locating downed pilots whose locations are relatively well known. In addition, today's small, inexpensive hand-held calculator has more computing power than that possessed by entire fleets 50 years ago. Since the 1940s, the basic search planning method for locating downed military aviators in wartime has been repeatedly extended to cope with situations encountered in modern civilian search and rescue efforts. However, until now, these extensions did not revisit the basic theory of search and try to apply it to today's search problems. Instead, previous efforts attempted to extend the solution for one very specific, limited, problem and make it work for other, much different problems. Often this was like trying to force a square peg into a round hole.

7.5 Comparing the SSPM and ISPM. This chapter will revisit the problem of optimally allocating search effort and will compare the previous method with the improved search planning method (ISPM) which uses the optimal search factors described in Chapter 6. In summary, the ISPM:

- Replaces the five fixed "safety factors" with continuous curves representing "optimal search factors" as functions of "cumulative relative search effort;"

- Works for point datums, line datums and area datums as well as ideal or poor search conditions. The assumption of a sensor modeled on the "inverse cube law" of visual detection with a known sweep width moving along equally spaced parallel tracks has been retained for ideal search conditions. For poor search conditions, it can be shown that the "poor search conditions" curves work for any sensor as long as the sweep width is known;

- Provides specific, quantitative guidance on how and where the available search effort should be employed so as to maximize search effectiveness, i.e., it answers the question of how to balance the amount of area searched against the level of coverage achieved in the searched area; and

- Provides measures of effectiveness, probability of success (POS) and cumulative POS, for evaluating the results of individual searches and series of searches.

7.5.1 Since both the SSPM and ISPM are based on the same theory, there should be some point of agreement between the two; that is, for some level of search effort, the two methods should recommend the same search area. Unfortunately, the SSPM does not use search effort to compute the recommended area, making it likely that the designers of this method made some assumptions about its value in order to make the method simpler. Most practicing search planners assume the recommended search area should be covered with a coverage factor of 1.0. Making this assumption allows a relative effort to be computed for the SSPM and provides an opportunity for comparing it to the ISPM.

7.5.2 The level of effort (Z) required to search an area with a coverage factor of 1.0 is exactly equal to the size of the area in square nautical miles. For example, the SSPM recommends the first search area be a square centered on datum which is just large enough to encompass a circle whose radius is 1.1 times the total probable error of position, E. The effort required to search such a square with a coverage factor of 1.0 is found by the formula:

$$Z = \text{Area} = (2.0 \times 1.1 \times E)^2 = (2.2 \times E)^2 = 4.84 \times E^2$$

To use the ISPM, it is necessary to compute the cumulative relative effort. Relative effort for a point datum is computed by the formula:

$$Z_r = \frac{Z}{f_{Zp}} = \frac{Z}{E^2}$$

Thus the relative effort (and cumulative relative effort) for the first search based on the SSPM's "safety factor" is:

$$Z_r = \frac{4.84 \times E^2}{E^2} = 4.84$$

The effort required to cover the SSPM's recommended second search area with a coverage factor of 1.0 is:

$$Z = (2.0 \times 1.6 \times E)^2 = 10.24 \times E^2$$

This means the relative effort is 10.24 making the cumulative relative effort for the first two searches:

$$Z_{rc} = Z_{r\text{-}1} + Z_{r\text{-}2} = 4.84 + 10.24 = 15.08$$

7.5.3 The relative effort and cumulative relative effort values for all five of the SSPM "safety factors" are listed in Table 7-1 below, assuming a coverage factor of 1.0 is used for the area defined by each "safety factor." The ISPM optimal search factors and other data of interest are also listed. (The optimal search factors were computed for Table 7-1 but may be found using Figures 6-3 and 6-4 of Chapter 6.) Note the very close agreement between the SSPM Safety Factor and the ISPM Optimal Search Factor (the second and fifth columns, respectively) for each of the first three searches.

Search Number	SSPM Safety Factor	Relative Effort	Cumulative Relative Effort	ISPM Optimal Search Factor	ISPM Cumulative Probability of Success	ISPM Optimal Coverage Factor
1	**1.1**	4.84	4.84	**1.09**	51.17%	1.02
2	**1.6**	10.24	15.08	**1.59**	82.70%	1.01
3	**2.0**	16.00	31.08	**2.02**	94.91%	0.98
4	**2.3**	21.16	52.24	**2.40**	98.61%	0.92
5	**2.5**	25.00	77.24	**2.74**	99.62%	0.83

Table 7-1

From the values in this table, it appears that the SSPM was designed to produce the optimal search area for a coverage factor of 1.0, at least for the first three searches. However, as a practical matter, this approach to search planning has some serious drawbacks.

7.5.4 There are basically two ways to create an optimal search plan. The first is to choose the desired probability of success (POS), find the minimum amount of search effort required to attain that POS and the area in which it should be applied, and then obtain the required amount of search effort. The second way to create an optimal search plan is to determine the amount of search effort readily available, then find the optimal area to search with it. In the first method, the search planner determines how much effort is required to get the desired result and then has to try to obtain the necessary resources. In the second method, the search planner determines what resources are available, then determines the best way to employ them. Both are practical approaches, but very often resource availability is determined by external factors independent of the search planner's desires. For this reason, the ISPM in assumes the amount of available effort is known and provides a way of determining the best way to use that effort. However, the graphs and formulas of the ISPM can be worked "backward" from a desired cumulative POS to the cumulative relative effort required, and then to the actual effort required in square nautical miles. The optimal search factor, search area, coverage factor, etc., can then be found in the usual fashion.

7.5.5 With the SSPM, an approach somewhat different from either of those just described was taken in order to simplify the method. Instead of a desired POS or cumulative POS, an assumption was made about the coverage factor to be used in all searches. The coverage factor was probably intended to always be 1.0 and the optimal area based on that assumption was supposed to be that produced by the "safety factor." As seen above, this is very nearly the case, especially for the first three searches. However, fixing the coverage factor for all searches severely limits the SSPM's flexibility, requiring the search planner to find specific amounts of search effort for each search or risk wasting search effort by not using it in the most optimal way.

7.5.6 If the search planner happens to have exactly the right amount of search effort available to cover the area recommended by the SSPM with a coverage factor of 1.0, then the SSPM works very well for at least the first three searches. Alternatively, if the search planner has the luxury of always being able to determine the amount of resources to commit to a search and decides to commit just the amount needed to cover the SSPM's recommended search area with a coverage factor of 1.0, then an optimal result *for the amount of effort actually expended* will be produced. However, in any other situation, the SSPM does not help the search planner make difficult decisions about the amount of area to cover with the available resources.

7.5.7 To illustrate the difficulty faced by the search planner, consider the situation where the available search effort for the first search is significantly different from than that required to cover the SSPM's recommended search area with a coverage factor of 1.0. In other words, the relative effort for the first search is significantly different from 4.84. The SSPM provides no guidance on how to apply such different levels of search effort. Two possible alternatives come immediately to mind.

- Alternative (1) - Cover as much area as possible with a coverage factor of 1.0. This will result in a search area significantly different from the one recommended.

- Alternative (2) - Cover the area recommended for the first search (based on a "safety factor" of 1.1) using the available search effort. This will result in a coverage factor significantly different from 1.0.

7.5.8 The following tables compare these two alternatives to the optimal search plan and to each other for various values of relative effort. As before, the following definitions apply:

POC:	Probability of Containment. The probability that the search object is in the search area.
POD:	Probability of Detection. The probability that the search object will be detected if it is in the search area.
POS:	Probability of Success. The probability that the search object will be found by covering the search area uniformly with the available search effort. POS = POC x POD.
Search Factor:	A value which, when multiplied by the total probable error of position (E) produces the "search radius" or

one-half the length of one side of the square search area. It corresponds to the “safety factor” used in the SSPM.

7.5.9 In all the tables below, it is assumed that an inverse cube law sensor moving along equally spaced parallel tracks under ideal search conditions is used within the search area. To keep the comparisons simple, a sweep width of 1.0 was assumed for all cases. The remaining values in the tables below were computed as follows.

- **Optimal** - The optimal search factor (f_s) was found using the computer program that generated the optimal search factor curves. The coverage factor (C) was then computed as the ratio of the relative effort to the "relative area" of a square just large enough to encompass a circle whose radius is f_s (i.e., with sides equal to 2 f_s.). That is,

[1] $$S = \frac{W}{C}$$

The track space was then computed as the sweep width divided by the coverage factor, or

[2] $$C = \frac{Z_r}{(2 f_s)^2}$$

The POC, POD and POS values were all computed with computer program modules based on standard algorithms.

- Alternative (1) - The coverage factor was assumed to be 1.0. The track space was computed using [2] above. The search factor was then computed by solving equation [1] for f_s

[3] $$f_s = \frac{1}{2}\sqrt{\frac{Z_r}{C}}$$

The POC, POD, and POS values were then computed using the same computer program modules as before.

- Alternative (2) - The search factor was assumed to be same as the first search "safety factor" recommended by the SSPM, or 1.1. The coverage factor, track spacing and other values were then computed just as they were for the optimal search factor.

7.5.10 In all three cases in each table, exactly the same amount of search effort was available. The first row in each table shows the optimal way to allocate the effort and the resulting POS. The second row shows what happens when the available effort is used with a coverage factor of 1.0. The third row shows what happens when the available effort is applied uniformly to the area found by using the SSPM "safety factor."

	Relative Effort = 1.0						
	Sweep Width	**Track Space**	**Coverage Factor**	**POC**	**POD**	**POS**	**Search Factor**
Optimal	1.0	1.65	0.60	30.38%	55.11%	**16.74%**	0.64
Alternative (1)	1.0	1.00	1.00	19.71%	78.99%	15.57%	0.50
Alternative (2)	1.0	4.84	0.21	64.76%	20.43%	13.23%	1.10

Table 7-2(a)

	Relative Effort = 2.5						
	Sweep Width	**Track Space**	**Coverage Factor**	**POC**	**POD**	**POS**	**Search Factor**
Optimal	1.0	1.22	0.82	48.44%	69.60%	**33.71%**	0.87
Alternative (1)	1.0	1.00	1.00	42.00%	78.99%	33.17%	0.79
Alternative (2)	1.0	1.94	0.52	64.76%	48.26%	31.25%	1.10

Table 7-2(b)

	Relative Effort = 4.84 *(same as that assumed by SSPM)*						
	Sweep Width	**Track Space**	**Coverage Factor**	**POC**	**POD**	**POS**	**Search Factor**
Optimal	1.0	0.98	1.02	63.97%	79.98%	**51.17%**	1.09
Alternative (1)	1.0	1.00	1.00	64.76%	78.99%	51.15%	1.10
Alternative (2)	1.0	1.00	1.00	64.76%	78.99%	51.15%	1.10

Table 7-2(c)

	Relative Effort = 10.0						
	Sweep Width	**Track Space**	**Coverage Factor**	**POC**	**POD**	**POS**	**Search Factor**
Optimal	1.0	0.77	1.30	80.50%	89.72%	**72.23%**	1.39

Alternative (1)	1.0	1.00	1.00	87.86%	78.99%	69.40%	1.58
Alternative (2)	1.0	0.48	2.07	64.76%	99.04%	64.14%	1.10

Table 7-2(d)

	Relative Effort = 20.0						
	Sweep Width	**Track Space**	**Coverage Factor**	**POC**	**POD**	**POS**	**Search Factor**
Optimal	1.0	0.61	1.64	92.20%	96.02%	**88.53%**	1.75
Alternative (1)	1.0	1.00	1.00	98.31%	78.99%	77.66%	2.24
Alternative (2)	1.0	0.24	4.13	64.76%	100.00 %	64.76%	1.10

Table 7-2(e)

7.6 Conclusions. The above tables compare the optimal ISPM and the alternative SSPM search plans in a way that shows the SSPM in the most favorable possible light. Even so, several conclusions about the first search around a point datum can be drawn from these tables.

- Using the optimal search factor from the ISPM always produces the best possible result, that is, the highest POS.

- If the optimal search factor is not known, searching as much area as possible at a coverage of 1.0 will produce better results than covering the SSPM's recommended search area at a higher or lower coverage. However, the POS will still be less than the optimal value.

- The fixed "safety factors" of the SSPM are not very useful in determining the best amount of area to cover with the available resources.

7.6.1 There are some other, more subtle, consequences and requirements of the SSPM. If the area computed from each successive "safety factor" is to be searched with a coverage factor of 1.0, then each search effort has to be larger than the previous one by exactly the right amount. That is, the second search effort would need to be 10.24/4.84 or 2.12 times as large as the first, the third would have to be 1.56 times as large as the second, etc. Any deviation from this sequence would produce less than optimal results. The ISPM, on the other hand, can handle any increase in the level of effort and almost any situation where the level of effort is smaller than that of the previous search. (A very high level of effort followed by and extremely low level will cause problems even for the ISPM. However, the level of effort on the latter search would have to be on the order of only about 10% of the level of the previous effort to make the ISPM produce a less than optimal result.) Unlike the SSPM, the ISPM gives the search planner the flexibility needed to deal with realistic variations in available search efforts during the progress of an extended search.

7.6.2 Although not shown in this paper, the ISPM may be applied to line and area datums as well as point datums. The SSPM was intended to be used with point datums alone. Therefore, there is no opportunity to compare the SSPM and ISPM for line and area datums.

7.7 Summary. To summarize, the advantages of the ISPM over the SSPM are:

- The ISPM always produces the highest possible cumulative probability of success (POS). (The SSPM provides an optimal result only when the available effort for each search is a certain special value in relation to the total probable error of position.)

- The ISPM is flexible with respect to the type of datum. The ISPM includes procedures, formulas and graphs that allow it to be applied to point datums, line datums and area datums. (The SSPM applies only to point datums.)

- The ISPM is flexible with respect to the amount of available search effort. It produces an optimal search factor tailored to the capabilities of the available search facilities. (The SSPM uses a fixed set of "safety factors" that require certain specific levels of effort to be obtained and used.)

- The ISPM is flexible with respect to the search conditions. Graphs are provided for both ideal and poor search conditions. (The SSPM considered only ideal search conditions.)

- The ISPM provides a measure of search effectiveness. Probability of Success (POS) and cumulative probability of success (POS_c) are used as criteria for optimizing the allocation of search effort and may be used by SAR Mission Coordinators (SMCs) as one factor in deciding on whether to continue searching. (The SSPM provides no measure of search effectiveness.)

- Finally, the ISPM is almost as easy to use, requiring only a few computations in addition to those already being done for the SSPM.

Afterword

The *rule of Bayes* and *Bayes's formula* were deliberately not mentioned in this paper in order to keep both the concepts and the computations as simple as possible. The basic principles were easier to explain without having to develop Bayes's formula for the benefit of those without a strong background in statistics. In addition, doing a full Bayesian update of a probability map requires considerably more computation and every effort was being made to minimize the amount of computation required.

The effect of the omitted computations amounts to omitting a *re-normalization* of the probability map following each search update. Such a Bayesian re-normalization would have restored the total POC for the possibility area to 1.0 after each search, regardless of how much searching was done. The more intuitive approach of "subtracting probability" from the possibility area until it fades away completely was deemed preferable for purposes of explaining the POC update process. It also made the computation of cumulative POS considerably easier and provided a direct and obvious connection between cumulative POS and remaining POC.

The omission of re-normalization computations in no way affects the development of optimal search plans. The optimal search radii will be the same regardless of whether probability maps are re-normalized after each search update. It should be noted that both the current and recently re-developed versions of the U. S. Coast Guard's Computer Assisted Search Planning (CASP) system use full Bayesian updates for probability map ***displays***, but compute cumulative POS by dividing the sum of all "pfail" (i.e., remaining probability) values on the individual simulated search objects by the total number of search objects in the simulation and subtracting the result from 1.0.

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Appendix A:
Statistical Tables

Table A-1
NORMAL DISTRIBUTION FUNCTION

$$P(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{1}{2}t^2}\, dt$$

x	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997

3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

Table A-2
ERROR FUNCTION

$$erf(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2}\, dt$$

x	**0.00**	**0.01**	**0.02**	**0.03**	**0.04**	**0.05**	**0.06**	**0.07**	**0.08**	**0.09**
0.0	0.0000	0.0113	0.0226	0.0338	0.0451	0.0564	0.0676	0.0789	0.0901	0.1013
0.1	0.1125	0.1236	0.1348	0.1459	0.1569	0.1680	0.1790	0.1900	0.2009	0.2118
0.2	0.2227	0.2335	0.2443	0.2550	0.2657	0.2763	0.2869	0.2974	0.3079	0.3183
0.3	0.3286	0.3389	0.3491	0.3593	0.3694	0.3794	0.3893	0.3992	0.4090	0.4187
0.4	0.4284	0.4380	0.4475	0.4569	0.4662	0.4755	0.4847	0.4937	0.5027	0.5117
0.5	0.5205	0.5292	0.5379	0.5465	0.5549	0.5633	0.5716	0.5798	0.5879	0.5959
0.6	0.6039	0.6117	0.6194	0.6270	0.6346	0.6420	0.6494	0.6566	0.6638	0.6708
0.7	0.6778	0.6847	0.6914	0.6981	0.7047	0.7112	0.7175	0.7238	0.7300	0.7361
0.8	0.7421	0.7480	0.7538	0.7595	0.7651	0.7707	0.7761	0.7814	0.7867	0.7918
0.9	0.7969	0.8019	0.8068	0.8116	0.8163	0.8209	0.8254	0.8299	0.8342	0.8385
1.0	0.8427	0.8468	0.8508	0.8548	0.8586	0.8624	0.8661	0.8698	0.8733	0.8768
1.1	0.8802	0.8835	0.8868	0.8900	0.8931	0.8961	0.8991	0.9020	0.9048	0.9076
1.2	0.9103	0.9130	0.9155	0.9181	0.9205	0.9229	0.9252	0.9257	0.9297	0.9319
1.3	0.9340	0.9361	0.9381	0.9400	0.9419	0.9438	0.9456	0.9473	0.9490	0.9507
1.4	0.9523	0.9539	0.9554	0.9569	0.9583	0.9597	0.9611	0.9624	0.9637	0.9649
1.5	0.9661	0.9673	0.9684	0.9695	0.9706	0.9716	0.9726	0.9736	0.9745	0.9755
1.6	0.9763	0.9772	0.9780	0.9788	0.9796	0.9804	0.9811	0.9818	0.9825	0.9832
1.7	0.9838	0.9844	0.9850	0.9856	0.9861	0.9867	0.9872	0.9877	0.9882	0.9886
1.8	0.9891	0.9895	0.9899	0.9903	0.9907	0.9911	0.9915	0.9918	0.9922	0.9925
1.9	0.9928	0.9931	0.9934	0.9937	0.9939	0.9942	0.9944	0.9947	0.9949	0.9951
2.0	0.9953									